Mykola Bratiichuk, Dorota Śliwińska
Silesian University of Technology, Institute of Mathematics

Queueing system with resume level

The report deals with a finite-capacity queueing system $M/GI/1/b$ in which the arrival epochs correspond to an ordinary Poisson process with parameter λ. The service time has a general distribution function $F(x)$. The system is supposed to have only $b-1$ waiting places and the service discipline is "first in-first out" supplemented by an additional condition. This condition includes the presence of *the resume level* a $(a < b)$ which operates as follows. Let $\xi_a(t,b)$ be the number of customers in the system at time t and $\xi_a(0,b) \in [0,b)$. Till the epoch $\tau_1(b) = \inf\{t > 0 : \xi_a(t,b) = b\}$ the system works as a standard $M/GI/1/b$ system. At the moment $\tau_1(b)$ the input of customers is shut down until the length of line decreases to the level a. At this moment, the input is allowed to resume. Introducing of this input flow control has two objectives. Firstly, we want to reduce the waiting time of the messages that are not rejected upon arrival. Secondly, at the instant $\tau_1(b)$ we can notify about the overfilling of the system so that the potential customers can be redirected to another server. In [1] one can find computational algorithm for $\pi_k(a,b)$—steady-state queue length distribution of such a system. In our opinion, the main drawback of that algorithm consists in exploiting the normalizing condition $\sum_{k=0}^{b} \pi_k(a,b) = 1$, and hence if we want to find, for example, $\pi_1(a,b)$ we need to compute **all** $\pi_k(a,b)$. Changing b or a means repeating our algorithm from the very beginning and few previous calculations can be used here. The last is especially important if we deal with optimization problems and the numbers a, b are used as control parameters.

In the report we propose another approach. It gives the explicit formulae for $\pi_k(a,b)$ and, what is essential, the functions taking part in these formulae don't depend on a, b. The convenient calculating algorithm for steady-state parameters is proposed. Some numerical illustrations are presented as well.

References

[1] H. Takagi, *Queueing Analysis*, Vol. 2, Elsevier Science Publishers B.V., The Netherlands, 1993.