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# Continuity of the metric projection in Banach function spaces

Let $E$ be a Banach space and $C$ be its nonempty subset. *The metric projection* $\mu(\cdot|C) : E \to 2^C$ is defined by the formula

$$\mu(f|C) = \big\{h \in C : \|f - h\|_E = e(f|C)\big\},$$

where $e(f|C) = \inf \big\{ \|f - g\|_E : g \in C\big\}$ is the *distance* from $f$ to the set $C$.

Every element in $\mu(f|C)$ is called a *best approximation* of $f$ with respect to $C$.

For any sequence $(C_n)$ of nonempty convex subsets of a Banach space $E$ we write $C_n \to C$ provided that $s \liminf C_n = w \limsup C_n$, where

$$s \liminf C_n = \big\{g \in E : \|g_n - g\|_E \to 0, \text{ for a } (g_n) \text{ with } g_n \in C_n,\ n \in \mathbb{N}\big\}$$

and

$$w \limsup C_n = \big\{g \in E : g_{n_k} \xrightarrow{w} g, \text{ for some } (n_k) \subset \mathbb{N} \text{ and } g_{n_k} \in C_{n_k}\big\}.$$

In approximation theory the problem of continuity of the metric projection is important from the point of view of applications. The continuity of the metric projections in Orlicz spaces were studied extensively by F. Zó, H.H. Cuenya, M. Li, Y. Teng and T. Wang.

We present some results on the properties of the metric projection in a certain class of Banach function spaces defined on finite nonatomic measure spaces. The obtained results applied to the Orlicz function spaces yield known results.

**Theorem.** *Let $E$ be a Banach function space on a finite nonatomic measure space $(\Omega, \Sigma, \nu)$ such that $E \subset L_1(\nu)$ and $E \neq L_1(\nu)$. Let $(C_n)$ be a sequence of nonempty convex subsets of $E$ that are closed in $L_1(\nu)$ and let $(f_n) \subset E$, $f \in E$. If $\|f_n - f\|_E \to 0$ and $C_n \to C \neq \emptyset$, then*

$$\big|e(f_n|C_n) - e(f|C)\big| \longrightarrow 0.$$

**Theorem.** *Let $E$ be a Banach function space on a finite nonatomic measure space $(\Omega, \Sigma, \nu)$ such that $E \subset L_1(\nu)$ and $E \neq L_1(\nu)$. Let $E$ be a rotund Banach function space with the $H$-property. Let $(C_n)$ be a sequence of nonempty convex subsets of $E$ that are closed in $L_1(\nu)$ and let $(f_n) \subset E$, $f \in E$.*

*If $\|f_n - f\|_E \to 0$ and $C_n \to C \neq \emptyset$, then*

$$\|\mu(f_n|C_n) - \mu(f|C)\|_E \longrightarrow 0.$$