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Approximate controllability

Notion of controllability is of great importance in mathematical control theory. It is closely related to the theory of minimal realisation and optimal control. Many fundamental problems of control theory such as pole-assignment, stabilisability and optimal control may be solved under assumption that system is controllable.

Roughly speaking, controllability means, that it is possible to steer a dynamic system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. To be brief, if a dynamic system is controllable, all modes of the system can be excited from the input. Controllability of a system permits the choice of state feedback resulting in desirable properties of closed-loop system.

There are many different definitions and criteria of controllability that depend both on state equation and constraints on the control signal. Most of the criteria, which can be met in literature, are formulated for finite dimensional systems. It should be pointed out that many unsolved problems still exit as far as controllability of infinite dimensional systems is concerned. In order to fill this gap, the present paper studies in detail the controllability of the second order undamped infinite dimensional system.

In the case of infinite dimensional systems two basic concepts of controllability can distinguished. There are exact and approximate controllability. This is strongly related to the fact that in infinite dimensional spaces there exist linear subspaces, which are not closed. Exact controllability enables to steer the system to arbitrary final state while approximate controllability means that system can be steered to arbitrary small neighbourhood of final state. In other words approximate controllability gives the possibility of steering the system to the states which form the dense subspace in the state space. Taking this into account it is obvious that exact controllability is essentially stronger notion than approximate controllability. In other words, exact controllability always implies approximate controllability. The converse statement is generally false. However, in the case of infinite dimensional systems exact controllability appears rather exceptionally. On the other hand, it should be stressed that in the case of finite dimensional systems notions of exact and approximate controllability coincide.

The main purpose of the paper is to formulate and prove necessary and sufficient conditions for approximate controllability of system under discussion. Using the spectral theory of linear unbounded operators, theory of semigroups and frequency domain conditions, it is proved that approximate controllability of second order undamped infinite dimensional system may be verified by means of controllability conditions for the corresponding simplified first order system. Finally, simple numerical example that illustrates the general theory is presented. In the example computable necessary and sufficient conditions for approximate controllability of linear distributed parameter dynamical system described by partial differential equation of hyperbolic type are presented.