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Approximation properties of certain positive linear operators

In this paper we examine approximation properties of general positive and linear operators L_n, $n \in N = \{1, 2, \ldots\}$, acting from the polynomial weighted space $C_p(Q)$ into $B_p(Q)$ and satisfying the condition $L_n(1; x) = 1$ for every $x \in Q$ and $n \in N$. Here $B_p(Q)$, $p \in N_0 = N \cup \{0\}$, is the set of all real-valued functions f defined on the interval $Q = [0, \infty)$ for which fw_p,

$$w_0(x) := 1, \quad w_p(x) := (1 + x^p)^{-1} \quad \text{if} \quad p \geq 1,$$

is bounded on Q. The norm in the space $B_p(Q)$ is defined by:

$$||f||_p = \sup_{x \in Q} w_p(x)\, |f(x)|.$$

Moreover $C_p(Q)$, $p \in N_0$, is the set of all $f \in B_p(Q)$ for which fw_p is a uniformly continuous function on Q.

Approximation properties of these operators L_n give the following

Theorem. *Let $p \in N_0$ be a fixed number. Then there exists a positive constant $M(p)$, depending only on p, such that for every $f \in C_p(Q)$ the following inequality holds*

$$w_p(x)|L_n(f; x) - f(x)| \leq M(p)\, \omega(f; \delta_n(x)),$$

for $x \in Q$ and $n \in N$, where $\omega(f)$ is the modulus of continuity of f and

$$\delta_n(x) = \left(L_n((t - x)^2; x)\right)^{\frac{1}{2}}.$$

In our paper we give also other approximation theorems.