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Bernstein-Durrmeyer type operators

In [1] J. L. Durrmeyer introduced an interesting modification of the Bernstein polynomials defined by

$$M_n(f;x) := (n+1)\sum_{k=0}^{n} p_{n,k}(x)\int_0^1 p_{n,k}(t)f(t)\,dt, \tag{1}$$

for $f \in C_{[0,1]}$, where $n \in N := \{1, 2, \ldots\}$ and $p_{n,k}(\cdot)$ are defined by

$$p_{n,k}(x) := \binom{n}{k} x^k(1-x)^{n-k}, \qquad 0 \le k \le n. \tag{2}$$

Approximation of continuous functions by Bernstein-Durrmeyer operator (defined by (1)), has been investigated by many authors.

In this paper we introduce the following class of operators in $C^r_{[0,1]}$.

Definition. Fix $r \in N_0$. We define a class of operators $M_{n,r}$ by the formula

$$M_{n,r}(f;x) := (n+1)\sum_{k=0}^{n} p_{n,k}(x)\int_0^1 p_{n,k}(t)\sum_{j=0}^{r}\frac{f^{(j)}(t)}{j!}(x-t)^j\,dt, \qquad x \in [0,1], \tag{3}$$

where $p_{n,k}(\cdot)$ are defined by (2).

Clearly, $M_{n,0}(f;x) = M_n(f;x)$ for $x \in [0,1]$, $n \in N$ and $f \in C_{[0,1]}$.

We shall study a relation between the rate of approximation by $M_{n,r}$ and the smoothness of the function f.

References

[1] J. L. Durrmeyer, *Une formule d'inversion de la transformée de Laplace: Applications à la théorie des moments*, Thèse de 3e cycle, Faculté des Sciences de l'Université de Paris, 1967.

[2] Z. Walczak, *Bernstein-Durrmeyer type operators*, Acta Mathematica Universitatis Ostraviensis 12 (2004), 65–72.