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Mathematical aspects of the initial-boundary value problems in nonlinear thermoelasticity of simple and non-simple materials

In this lecture we report on different coupled thermoelastic systems. They are models for the description of elastic, heat conductive media. We will consider a coupled second-order hyperbolic system describing the thermoelasticity of simple materials and a coupled parabolic-hyperbolic systems of thermoelasticity of non-simple materials. It is known that the classical thermoelasticity theory (hyperbolic-parabolic) leads to a parabolic differential equation for the temperature distribution in rigid heat conductors. This implies that thermal perturbations are felt instantaneously in every part of the body. Although, at first sight, this outcome seems to contradict the physical intuition, it can be justified by resorting to the fact that molecular motion, which places a crucial part in transport phenomena, is vary rapid except at extremely low temperatures. Hence finite velocity of propagation for thermal perturbations is usually non-observable unless experiments are performed in some neighborhood of absolute zero as in the case of liquid helium. In fact, thermal waves, commonly known as second sound, are detected in some metals cooled approximately down to 20°K. In our lecture we consider the theory of thermoelasticity by considering the temperature-rate-dependence and assigning an appropriate constitutive function to the entropy flux. Such a theory leads to a hyperbolic differential equation for thermal perturbations different from the equation describing propagation of thermal perturbation in classical thermoelasticity which is parabolic one. We consider also, the nonlinear hyperbolic-parabolic system of coupled partial differential equation of fourth order describing the thermoelasticity of non-simple materials. We are interested in both the lineralized and the non-linear systems looking for the description of the asymptotic behavior of smooth solutions, for smoothing effects of the systems and specifically for the non-linear systems for the global existence in time of solutions. For hyperbolic systems it is well known that in many cases locally existing smooth solutions tend to develop singularities in finite time. The basic problem for the system in question here is whatever and in which way the added damping by heat conduction or viscosity will assure the global existence of solutions. From this point of view we investigate the global existence in time for large data in nonlinear thermoviscoelasticity.

References

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