Local triviality and Borsuk-Ulam type theorems for actions of compact quantum groups

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Definition

A principal bundle (X, π, M, G) is said to be *locally trivial*,
if for every $p \in M$ there exists a neighbourhood U
and a G-equivariant homeomorphism $\varphi : U \times G \to \pi^{-1}(U)$
such that $\pi \circ \varphi : U \times G \to U$ is the canonical projection.

cpc $\perp$ machinery

Definition

Let A and B be C*-algebras, and let $\varphi : A \to B$ be a completely positive map. We say that φ has *order zero* if for every a and b in A, we have $\varphi(a) \perp \varphi(b)$ whenever $a \perp b$.

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Let A and B be C-algebras. There is a bijection between completely positive contractive order zero maps $A \to B$ and *-homomorphisms $C_0((0,1]) \otimes A \to B$.*

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When A and B are unital there is a bijection between cpc $\perp$ maps and unital *-homomorphims $\mathcal{C}A \to B$. This result extends to G-equivariant maps. Winter and Zacharias developed a functional calculus for c.p.c. $\perp$ maps as well.

Local triviality for actions of compact quantum groups

Definition

Let A be a unital C*-algebra with an action δ of a compact quantum group $\mathbb{G}$. For $d \geq 0$, we say that the system (A, $\mathbb{G}$, δ) has *triviality dimension at most* d, written $\dim_{\mathrm{triv}}^{\mathbb{G}}(A) \leq d$, if there exist completely positive contractive equivariant order zero maps $\varphi_0, \ldots, \varphi_d \colon C(\mathbb{G}) \to A$ satisfying $\sum_{j=0}^{d} \varphi_j(1) = 1$.

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Let $\dim_{\mathrm{triv}}^{\mathbb{G}}(A) = 0$. Then we have an equivariant c.p.c. order zero map $\varphi : C(\mathbb{G}) \to A$ and such that $\varphi(1) = 1$. One can prove that such a φ is a unital *-homomorphism and therefore $(A, \mathbb{G}, \delta)$ is a trivial bundle.

Reformulation of classical local triviality

Definition (Schwarz genus)

The *Schwarz genus* of a free G-space X, denoted by $g_G(X)$, is the smallest number n such that X can be covered with open G-invariant subsets $U_0, \ldots, U_n$ with the property that for every $0 \leq i \leq n$ there exists a G-equivariant map $U_i \to G$.

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Definition (G-index)

Let X be a free G-space. We define the *G-index* of X by

$$ind_G(X) := \min\{n \geq 0 : \exists G{-}map\ X \to E_n G\},$$

where $E_n G := \underbrace{G * \ldots * G}_{n+1}$.

Reformulation of classical local triviality (II)

Theorem

Let X be a compact Hausdorff space equipped with a free action of a compact Hausdorff group G. Then

$$g_G(X) = ind_G(X) = dim^G_{\mathrm{triv}}(C(X))$$

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For the second equality, one need to use the fact $E_n G$ is a subspace of $\mathcal{C}G^{n+1}$ and then use the Winter-Zacharias theorem for c.p.c. order zero maps.

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Definition (D. Ellwood)

Let $\delta : A \to A \otimes C(\mathbb{G})$ be an action of a compact quantum group $\mathbb{G}$ on a unital C*-algebra A. We say that δ is free iff $\{(x \otimes 1)\delta(y) : x, y \in A\}^{cls} = A \otimes C(\mathbb{G})$.

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Of course, $\Delta : C(\mathbb{G}) \to C(\mathbb{G}) \otimes C(\mathbb{G})$ is free by definition and for a fixed $x \in C(\mathbb{G})$ we have

$$1_{C(\mathbb{G})} \otimes x \underset{\varepsilon}{\approx} \sum_{k=1}^{m} (y_k \otimes 1_{C(\mathbb{G})})\Delta(z_k).$$

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where we used the functional calculus for c.p.c. order zero maps and the equivariance of φ_j's.
This shows that $1_A \otimes x$ belongs to B, and hence $B = A \otimes C(\mathbb{G})$ and we conclude that δ is free.

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Theorem (Borsuk-Ulam)

Let n be a positive natural number. There is no $\mathbb{Z}/2\mathbb{Z}$-equivariant continuous map $f : S^n * \mathbb{Z}/2\mathbb{Z} \to S^n$.

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Conjecture (Baum-Dąbrowski-Hajac)

Let X be a compact Hausdorff topological space equipped with a continuous free action of a non-trivial compact Hausdorff group G. Then, for the diagonal action of G on $X * G$, there does not exists a G-equivariant continuous map $f : X * G \to X$.

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Let A be a C-algebra and let G be a compact Hausdorff group with torsion. Then there is no G-equivariant map $A \to A \circledast C(G)$.*

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Theorem (Dąbrowski, Hajac, Neshveyev)

Let A be a unital C-algebra with a free action of a non-trivial compact quantum group $\mathbb{G}$. Then, if $C(\mathbb{G})$ admits a character that is not convolution idempotent, there is no $\mathbb{G}$-equivariant *-homomorphism $A \to A \circledast^{\delta} C(\mathbb{G})$.*

Borsuk-Ulam type result for locally trivial actions

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Proposition

Let $\mathbb{G}$ be a compact quantum group, let A be a unital C-algebra, and let δ be an action of $\mathbb{G}$ on A. Let I be an $\mathbb{G}$-invariant ideal in A, and denote by $\overline{\delta}$ the action induced by δ on the quotient A/I. Then*

$$\dim_{\mathrm{triv}}^{\mathbb{G}}(A/I) \leq \dim_{\mathrm{triv}}^{\mathbb{G}}(A).$$

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Theorem (Borsuk-Ulam type)

Let A be a unital C-algebra with an action of a compact Hausdorff group G and let $dim^{G}_{\mathrm{triv}}(A) < \infty$. Then there is no G-equivariant map $A \to A \circledast C(G)$.*