RANK-TWO MILNOR IDEMPOTENTS FOR THE MULTIPULLBACK QUANTUM COMPLEX PROJECTIVE PLANE

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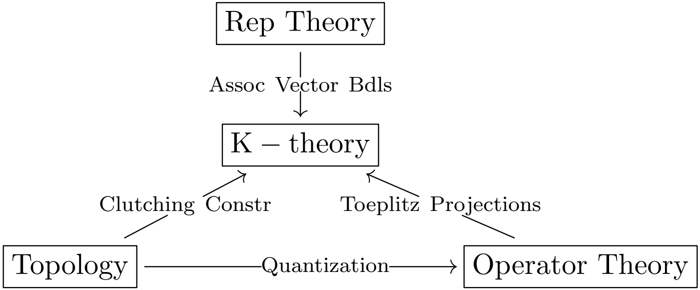
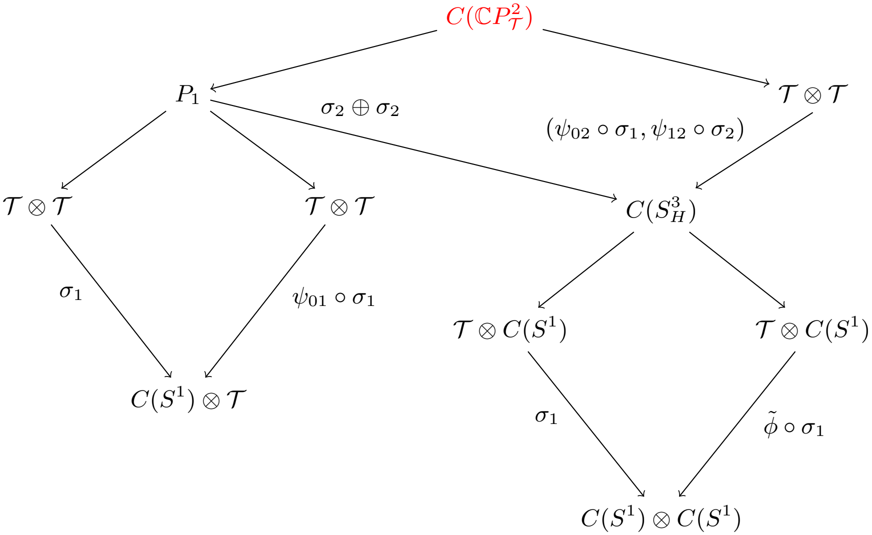
Un travail conjoint avec
Carla Farsi, Tomasz Maszczyk and Bartosz Zieliński.

Road map


Rep Theory
Assoc Vector Bdls
K – theory
Clutching Constr
Toeplitz Projections
Topology
Quantization
Operator Theory

Multipullback quantum complex projective



Multipullback quantum spheres S_H^{2N+1}

$C(S_H^{2N+1})$ is the C*-subalgebra of $\prod_{i=0}^{N} \mathcal{T}^{\otimes i} \otimes C(S^1) \otimes \mathcal{T}^{\otimes N-i}$ defined by the compatibility conditions prescribed by the following diagrams ($0 \leq i < j \leq N$, $\otimes$-supressed):

$$\mathcal{T}^i C(S^1) \mathcal{T}^{N-i} \xrightarrow{\sigma_j} \mathcal{T}^i C(S^1) \mathcal{T}^{j-i-1} C(S^1) \mathcal{T}^{N-j} \xleftarrow{\sigma_i} \mathcal{T}^j C(S^1) \mathcal{T}^{N-j}.$$

Here $\sigma_k := \mathrm{id}^k \otimes \sigma \otimes \mathrm{id}^{N-k}$ with domains and codomains determined by the context.

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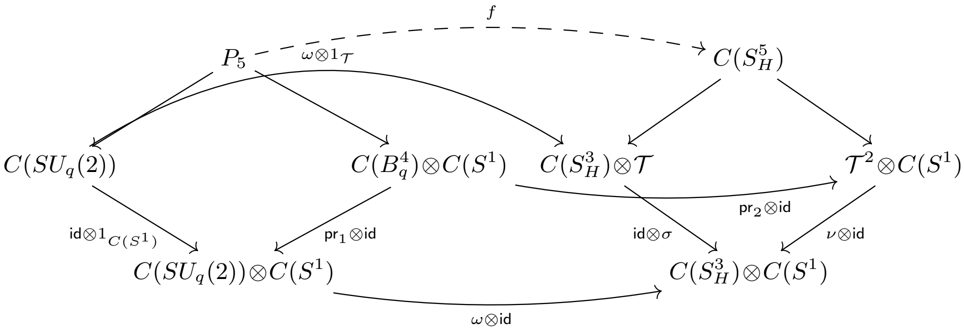
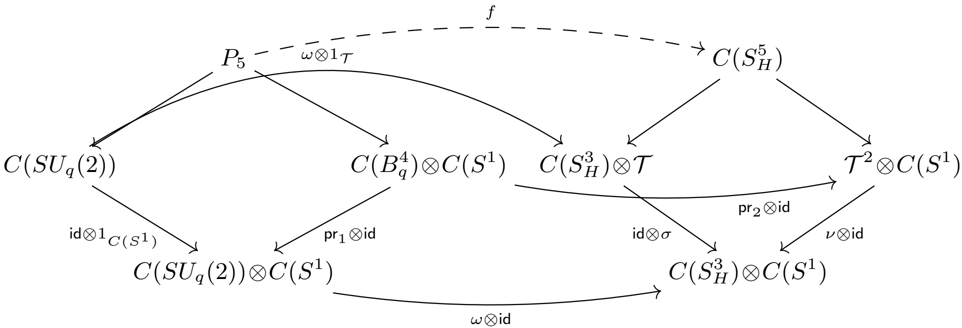
We equip all C*-algebras in the diagrams with the diagonal actions of $U(1)$. Since all morphisms in the diagrams are $U(1)$-equivariant, we obtain the diagonal $U(1)$-action on $C(S_H^{2N+1})$.

Reducing to the quantum-group case





Lemma

*The above *-homomorphisms are $U(1)$-equivariant, and the induced *-homomorphisms on fixed-point subalgebras yield isomorphisms on K-groups.*

Modules associated to piecewise cleft coactions

Let $\mathcal{H}$ be a Hopf algebra, let

$$\begin{array}{ccccc} & & \mathcal{P} & & \\ & \swarrow & & \searrow & \\ \mathcal{P}_1 & \xrightarrow[\tilde{\pi}_1]{} & \mathcal{P}_{12} & \xleftarrow[\tilde{\pi}_2]{} & \mathcal{P}_2 \end{array}$$

be a one-surjective pullback diagram of $\mathcal{H}$-comodule algebras, and let $\gamma_i : \mathcal{H} \to \mathcal{P}_i$ be cleaving maps.

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Theorem

If V is a finite-dimensional left $\mathcal{H}$-comodule, then the associated left $\mathcal{P}_{12}^{\mathrm{co}\,\mathcal{H}}$-module $\mathcal{P}\Box V$ is the Milnor module for the automorphism of $\mathcal{P}_{12}^{\mathrm{co}\,\mathcal{H}} \otimes V$ given by

$$b \otimes v \longmapsto b(\tilde{\pi}_1 \circ \gamma_1)(v_{(-2)})(\tilde{\pi}_2 \circ \gamma_2)^{-1}(v_{(-1)}) \otimes v_{(0)}.$$

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Applying this theorem to the $SU_q(2)$-prolongation of P_5 is a key to the main result.

Main result

Definition

Let $k \in \mathbb{Z}$. We call the left $C(\mathbb{C}P^2_{\mathcal{T}})$-module

$$L_k := \{a \in C(S^5_H) \mid \forall\, \lambda \in U(1) : \alpha_\lambda(a) = \lambda^k a\}$$

the section module of the associated line bundle of winding number k.

Theorem

The group $K_0(C(\mathbb{C}P^2_{\mathcal{T}}))$ is freely generated by elements

$$[1], \quad [L_1] - [1], \quad [L_1 \oplus L_{-1}] - [2].$$

Furthermore, $L_1 \oplus L_{-1} \cong C(\mathbb{C}P^2_{\mathcal{T}}) \oplus C(\mathbb{C}P^2_{\mathcal{T}})e$. Here $e \in C(\mathbb{C}P^2_{\mathcal{T}})$ is an idempotent such that $C(\mathbb{C}P^2_{\mathcal{T}})e$ cannot be realized as a finitely generated projective module associated with the $U(1)$-C-algebra $C(S^5_H)$ of Heegaard quantum 5-sphere.*

Tentative plan of conferences

New Geometry of Quantum Dynamics planned conferences:

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- The Banach Center, Warsaw, 15 January – 19 January 2018 (approved and funded)
- The Fields Institute, Toronto, 22 July – 16 August 2019 (pending approval)