A semilinear curl-curl problem in $\mathbb{R}^3$

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Abstract

We look for nontrivial solutions to the semilinear problem

$$\nabla \times \nabla \times u = f(x, u) \text{ in } \mathbb{R}^3,$$

where $f = \nabla_u F$ is $\mathbb{Z}^3$-periodic in x. We give sufficient conditions on the nonlinearity which provide a least energy solution and infinitely many $\mathbb{Z}^3$-distinct solutions.

The growth and asymptotic behaviour of the nonlinearity are described by an N-function which allows us to consider other model problems than the classical power type or double-power type.

After building the proper function space where to look for solutions and showing its main characteristics, we develop an abstract critical point theory, providing results that we use to solve our equation and may be applied to other problems.

The main difficulties are due to working in an unbounded domain and the infinite dimension of the kernel of $u \mapsto \nabla \times u$, i.e. the space of gradient vector fields. We overcome the former using a concentration-compactness argument.

At the end, we show how to solve Schrödinger's Equation using the abstract critical point theory.

This talk is based on a joint paper with Jarosław Mederski and Andrzej Szulkin.