MINICOURSE: BANACH REPRESENTATIONS OF DYNAMICAL SYSTEMS WARSAW, APRIL 2014

1. Abstract

Banach representations of dynamical systems is a relatively new theory already having several important applications. In the present course we study some hierarchies of topological dynamical systems and topological groups coming from Banach space theory. This allows to find new links between many different research lines. Among others: abstract topological dynamics, geometry of Banach spaces and the theory of Polish topological groups.

Like topological groups, compact dynamical systems, can be represented on Banach spaces. We study dynamical analogs of Eberlein, Radon-Nikodým and weakly Radon-Nikodým compacta; that is the classes of dynamical systems which can be represented on reflexiive, Asplund and Rosenthal Banach spaces. They correspond to important classes of compact metrizable dynamical systems: weakly almost periodic (WAP), hereditarily nonsensitive (HNS) and tame.

This approach naturally extends some classical research themes and at the same time opens new and sometimes quite unexpected directions. One of the examples is a connection between the lack of chaotic behavior (lack of "butterfly effects") of a dynamical system (HNS systems) and the existence of weak-star continuous representations on the dual of Asplund Banach spaces. The topological concept of the *fragmentability* (originally coming from Banach spaces) and the famous factorization theorem of Davis-Figiel-Johnson-Pelczyński are the main tools in the present theory.

We provide the necessary background. Besides some new results we give soft geometric proofs of several classical results (like: Teleman's regular representations of topological groups, Ellis and Ellis-Lawson theorems; Helmer's theorem about WAP functions; Ryll-Nardzewski's fixed point theorem, etc.). At the same time we discuss perspectives of the theory and pose several open questions.

During this course we will expose some results and ideas from a survey work (joint with E. Glasner) published recently in the new volume of "Recent Progress in General Topology III", Springer, 2013 (eds. K.P. Hart, J. van Mill and P. Simon). You can download this work, as well as, some related papers, from my homepage: www.math.biu.ac.il/~megereli.

2. About the lecturer

Professor Michael Megrelishvili received his doctorate in 1988 (Tbilisi State University). In 1988-1991 he was a senior researcher at the Mathematical Institute of Georgian Academy of Sciences. Since 1992 Megrelishvili works at the department of Mathematics in Bar-Ilan University (Ramat-Gan, Israel). He is currently an Associate Professor at the same university. His main research interests are: topological dynamical systems and topological groups. The present research theme is: "Representations of dynamical systems on Banach spaces". This research is supported by Israel Science Foundation Grant (2013-2017), joint with Prof. E. Glasner (Tel Aviv University). Previously this research has been supported by Israel-USA Binational Science Foundation (2007-2011) when Prof. V. Uspenskij (Ohio University) was also one of the co-researchers.

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