A series of lectures by

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Talk 1: p-Laplacian and basic properties

We introduce the p-Laplacian as minimizers of the p-Dirichlet energy. We introduce the necessary spaces and prove existence of solutions. Moreover, we discuss the maximal principle and the convex hull property.

Talk 2: Regularity of p-harmonic function

We prove higher regularity of p-harmonic functions. In particular, we prove $C^{1,\alpha}$ regularity and suitable decay estimates. Moreover, we discuss the regularity of p-harmonic functions in the plane in more detail.

Talk 3: p-Stokes system

We introduce the equations governing the motion of an incompressible generalized Newtonian fluid. A first version of Korn's inequality and the negative norm theorem are introduced.

Talk 4: Maximal function and covering theorems

The Hardy-Littlewood maximal operator M plays a fundamental role in harmonic analysis. We explain the basic properties of M and introduce the necessary covering theorems.

Talk 5: Korn and Bogovskiĭ

In the context of fluid mechanics it is often necessary to work with symmetric gradients, which are just the symmetric part of the gradient of a velocity field. Since Sobolev functions are defined by gradients, but the equation of motion give control of the symmetric gradient, it is necessary to control the gradients by the symmetric gradients. This is known as Korn's inequality. We talk about different aspect of this inequality.

Talk 6: Non-linear Calderón-Zygmund theory

The theory of singular integral operator is a very strong tool from harmonic analysis. Among other things it allows to transfer the regularity of the data to the solution of a linear PDE. If for example $-\Delta u = f$, then regularity of f is similar to $\nabla^2 u$, since $f \mapsto \nabla^2 u$ is a singular integral operator. This idea was extended by Iwaniec to the situation of the p-Laplacian. We explain this principle.

Talk 7: Lipschitz truncation

It is often very useful to approximate Sobolev functions by smooth functions. Usually this is done by convolution. However, it is sometimes necessary that the approximation differs from the original function only on a small set. The Lipschitz truncation technique allows to find such approximate functions. It is closely related to the Calderón-Zygmund decomposition of Sobolev functions.

Talk 8: Parabolic Lipschitz truncation

We extend the Lipschitz truncation technique to the parabolic setting.

Talk 9: Finite elements – a priori estimates
We provide a priori estimates for the finite element solutions of the p-Laplacian.

Talk 10: Finite elements – A posteriori estimates
We discuss a posteriori estimates for the finite element solutions of the p-Laplacian. We prove optimalital rates for the algorithm.