Bifurcations and Auerbach bases

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Let V be a Banach space of finite dimension. We say that a basis $v_1, v_2, ..., v_n \in V$ has Auerbach property if $|v_i| = |v_i^*| = 1$ for all $i = 1, 2, \ldots, n$. Existence of such a basis was proved by Auerbach in 1930. This property was studied by many authors in functional analysis and by specialists in convex bodies. Quite recently, in 2005, Plichko has shown that for a given Banach space there exist at least two essentially different Auerbach bases. Next, Pełczyński conjectured that in a Banach space of dimension n there exist at least n essentially different Auerbach bases. We will show that indeed the conjecture is true, but in fact there exists $\frac{n(n-1)}{2}$ such bases, and in generic cases there is an exponential lower bound. The proof is based on singularity theory, namely an estimation of a number of bifurcation points by means of Lusternik-Schnilermann category.

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