

The Łojasiewicz exponent of non-negative and non-degenerate polynomials at infinity

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Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a polynomial, which is non-negative outside some compact set. We assume that $f^{-1}(0)$ is not compact and f is non-degenerate in the Kushnirenko sense at infinity. We give a formula for the Łojasiewicz exponent $\mathcal{L}_\infty(f)$ in terms of its Newton polyhedron. By definition $\mathcal{L}_\infty(f)$ is the least exponent $\nu \in \mathbb{R}$ such that the inequality

$$|f(x)| \geq C \operatorname{dist}(x, f^{-1}(0))^\nu,$$

holds in a neighbourhood of the level set $f^{-1}(0)$ at infinity i.e. on the set

$$\{x \in \mathbb{R}^n : \operatorname{dist}(x, f^{-1}(0)) < \varepsilon, |x| \geq R\}$$

for some constant $\varepsilon, R, C > 0$.

This is a joint work with Adam Różycki.