G_2-Geometry in Contact Geometry of Second Order

Keizo YAMAGUCHI

Classical theory for systems of the first order partial differential equations for a scalar function can be rephrased as the submanifold theory of contact manifolds (geometric first order jet spaces). In the same spirit, we have developed the geometric theory of systems of partial differential equations of second order for a scalar function as the **Contact Geometry of Second Order**, following E.Cartan. We have formulated the submanifold theory of second order jet spaces as the geometry of PD manifolds $(R; D^1, D^2)$ of second order. Moreover we have established the First and Second Reduction Theorems for $(R; D^1, D^2)$. In this talk, we will revisit G_2-Geometry in Contact Geometry of Second Order, where we can construct for each Exceptional Simple Lie Algebra, the model system of partial differential equations of second order, whose infinitesimal contact transformations is isomorphic with the given simple Lie algebra. Starting point of this geometry is the following historical example by E.Cartan, whose symmetry algebra of the infinitesimal contact transformations is isomorphic to the 14-dimensional exceptional simple Lie algebra of type G_2:

Overdetermined (involutive) system :

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{3}\left(\frac{\partial^2 z}{\partial y^2}\right)^3, \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{1}{2}\left(\frac{\partial^2 z}{\partial y^2}\right)^2.$$

We will show the explicit construction for the B_3 case.

K.Yamaguchi, Department of Mathematics,, Faculty of Science,, Hokkaido University,, Sapporo 060-0810,, Japan, E-mail yamaguch@math.sci.hokudai.ac.jp