

# Singularities of secant maps from a global viewpoint

M.C. Romero Fuster

August 22, 2017

The local properties of secant maps associated to curves in 3-space were first analyzed by J.W. Bruce [1] who proved that, for generic pairs of curves, such maps are locally stable and thus may only have isolated cross-cap points. We consider here the case of two closed plane curves and analyze its singularities from the local and multi-local viewpoints, providing their geometrical characterization up to codimension 2. It can be seen, as a consequence of Thom's fundamental transversality lemma, that the secant map of a generic pair of closed plane curves is a stable map from the torus to the plane. In the particular case  $\alpha = \beta$  we get that for most rigid motions  $\phi$  on the plane the pair  $(\alpha, \phi \cdot \alpha)$  is a generic couple of curves too.

From a global viewpoint, we show that *the number of connected components of the singular set of the secant map of a generic pair of closed convex curves with respective Whitney indexes  $n$  and  $m$ , is exactly twice the maximum common divisor  $\mu_{n,m}$  of  $n$  and  $m$ . Moreover, all the singular curves are of type  $(\frac{n}{\mu_{n,m}}, \frac{m}{\mu_{n,m}})$ .*

As a consequence we get that *given a convex curve  $\alpha$  with Whitney index  $n$ , the secant map of pair  $(\alpha, \phi \cdot \alpha)$  (where  $\phi$  is any rigid motion such that  $(\alpha, \phi \cdot \alpha)$  is a generic pair of curves) has exactly  $n$  singular curves, all of them being toric curves of type  $(1, 1)$ . In the non convex case, the singular set of the secant map may also have a certain number (bounded by the sum of the total numbers of inflection points in both curves) of homologically trivial closed toric curves. This work has been jointly developed with L. Sanhermelando [5].*

The image of the singular set of the secant map is a collection of closed plane curves with isolated cusps corresponding to pairs of points having parallel tangents and equal (non vanishing) curvature. We call it the *secant caustic*. Some global results concerning this set, obtained in collaboration with W. Domitrz and M. Zwierzyński, are obtained in [2].

We finally describe some global properties of the secant maps associated to curves in 3-space ([3], [4]).

## References

- [1] J. W. Bruce, Generic Space Curves and Secants. *Proc. Royal Soc. Edimburgh*
- [2] W. Domitrz, M. C. Romero Fuster, M. Zwierzyński, The geometry of the Secant Caustic (preprint).
- [3] D. Hacon, C, Mendes de Jesus, M. C. Romero Fuster, Graphs of stable maps from closed orientable surfaces to the 2-sphere. *J. Singul.* 2 (2010), 6780.
- [4] J. J. Nuño and M. C. Romero Fuster, Global bitangency properties of generic closed space curves. *Math. Proc. Cambridge Philos. Soc.* 112 (1992).
- [5] M. C. Romero Fuster and L. Sanhermelando, Singularities of secant maps on closed plane curves. Proceedings of the workshop “Singularities in Generic Geometry and its Applications – Kobe-Kyoto 2015 (Valencia IV)”. *Advanced Studies in Pure Mathematics*. To appear.