Simplifying indefinite fibrations on 4–manifolds

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A broken Lefschetz fibration (BLF, for short) is a smooth map of a closed oriented 4–manifold onto a closed surface whose singularities consist of Lefschetz critical points together with indefinite folds (or round singularities). Such a class of maps was first introduced by Auroux–Donaldson–Katzarkov (2005) in relation to near-symplectic structures. In this talk, we give a set of explicit moves for BLFs, and give an elementary and constructive proof to the fact that any map into the 2–sphere is homotopic to a BLF with embedded round image. We also show how to realize any given null-homologous 1–dimensional submanifold with prescribed local models for its components as the round locus of a BLF. These algorithms allow us to give a purely topological and constructive proof of a theorem of Auroux–Donaldson–Katzarkov on the existence of broken Lefschetz pencils with embedded round image on near-symplectic 4–manifolds. We moreover establish a correspondence between BLFs and Gay–Kirby trisections of 4–manifolds, and show the existence of simplified trisections on all 4–manifolds. This is a joint work with R. İnanç Baykur (University of Massachusetts).