Covariant and observer-independent approach to geometric optics in GR

Mikołaj Korzyński (CFT PAN Warsaw)

in collaboration with

Jarosław Kopiński (University of Warsaw)
Michele Grasso (CFT PAN, Warsaw)
Julius Serbenta (CFT PAN, Warsaw)

IX International Meeting on Lorentzian Geometry
17th-24th June 2018
Banach Center, Warsaw

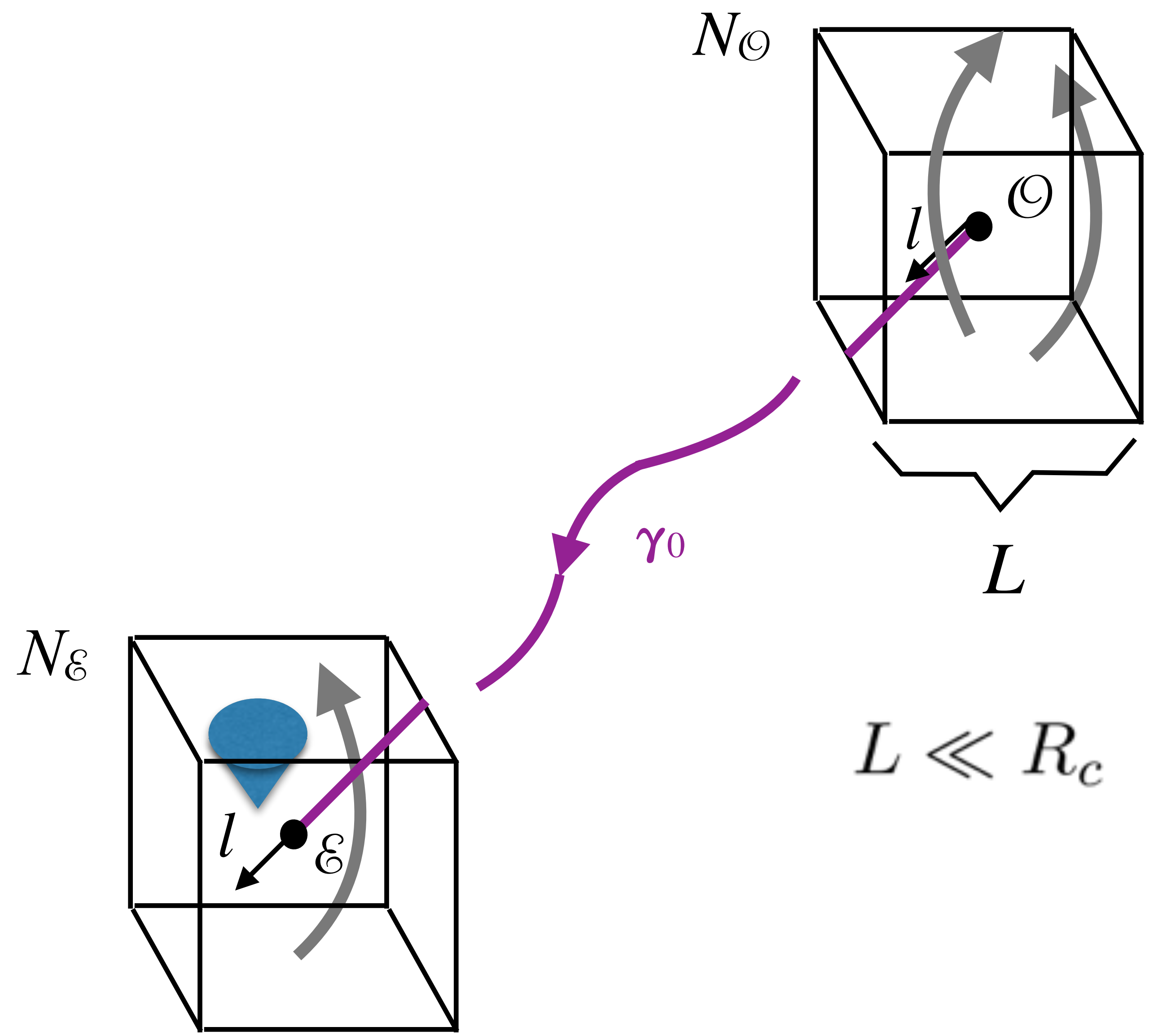
Papers

Based on:

- M.K., J. Kopiński *"Optical drift effects in general relativity"*, JCAP 03 (2018) 012, e-print: 1711.00584 [gr-qc]
- M.K., M. Grasso, J. Serbenta *"Geometric optics in general relativity using bi-local operators"*, in preparation

NCN project SONATA BIS No 2016/22/E/ST9/00578
"Local relativistic perturbative framework in hydrodynamics and general relativity and its application to cosmology"

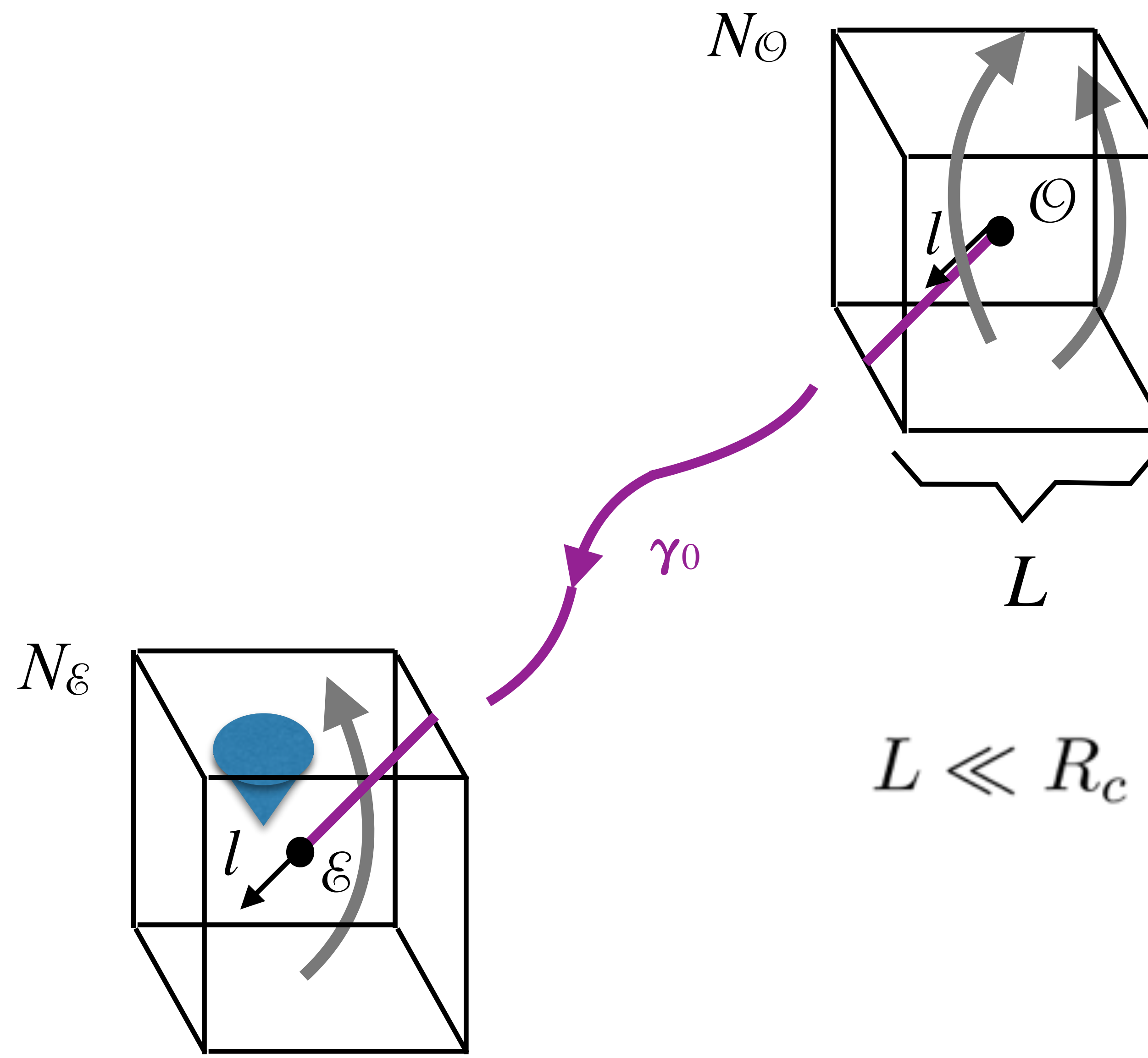
Motivation



$L \ll R_c$

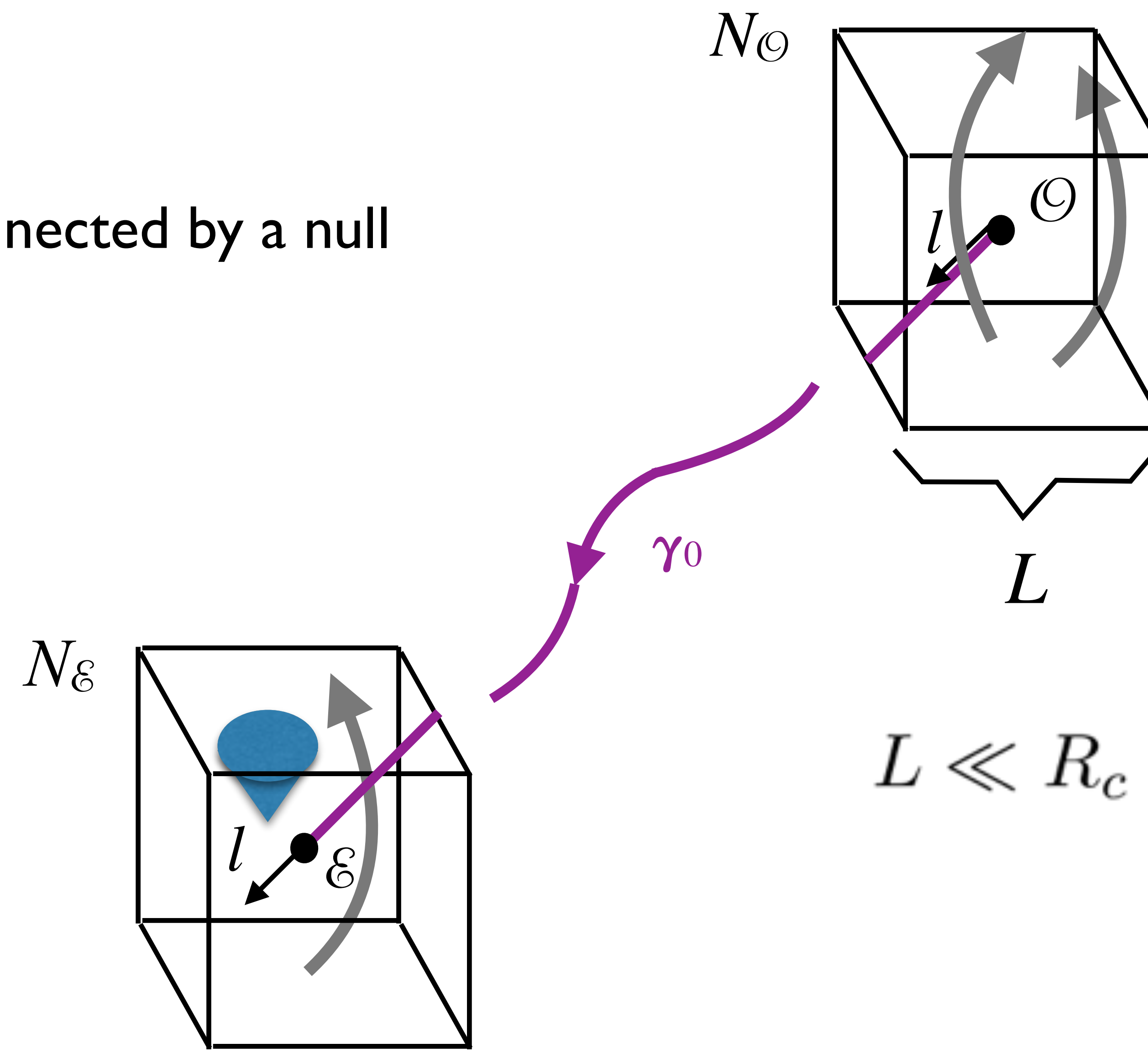
Motivation

- Spacetime with *any* Lorentzian metric



Motivation

- Spacetime with *any* Lorentzian metric
- Two distant, small regions of spacetime, connected by a null geodesics



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- Spacetime with *any* Lorentzian metric
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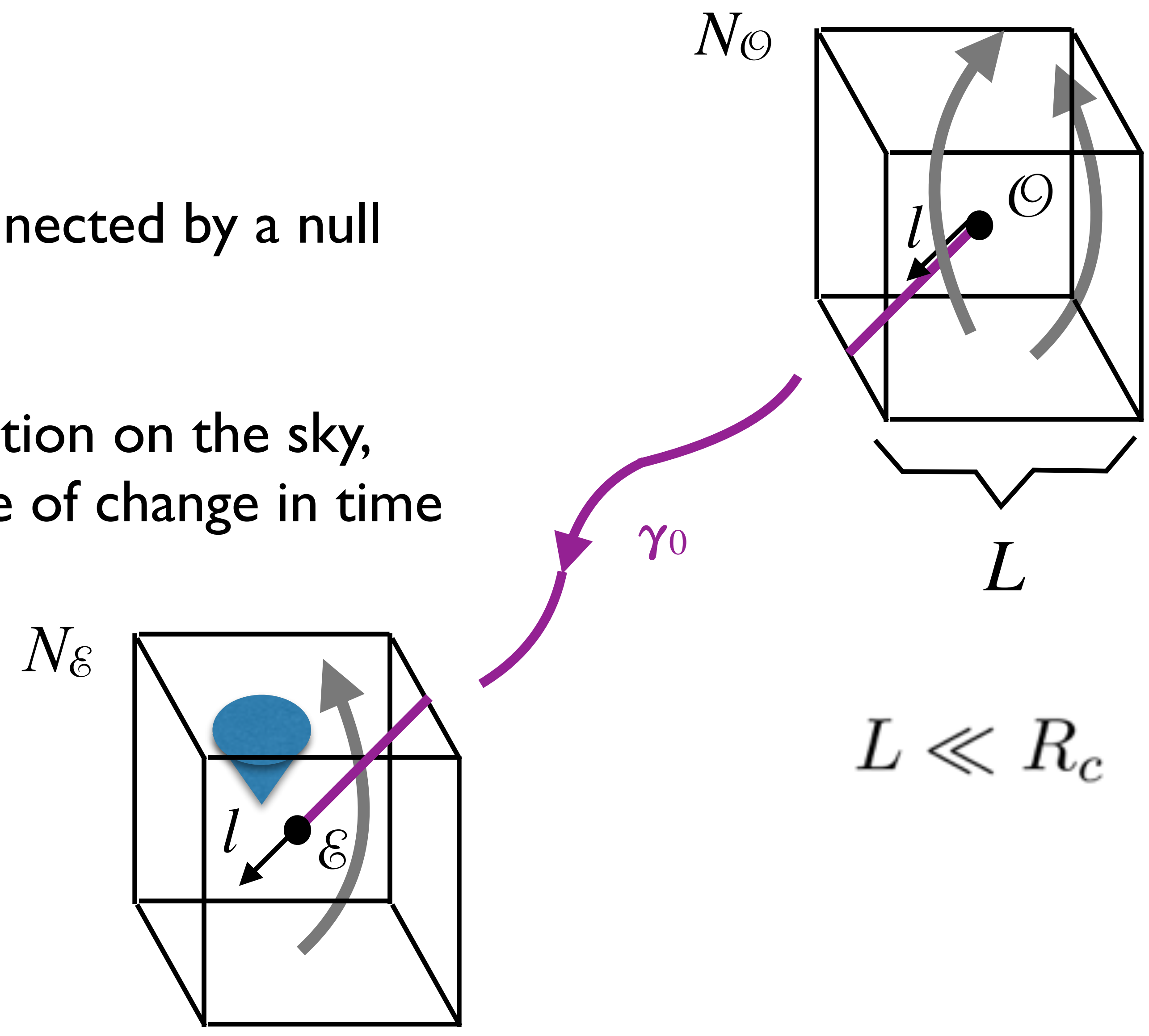
- Observers measure the time of arrival, position on the sky, redshift, image size and distortion, their rate of change in time (drift) …



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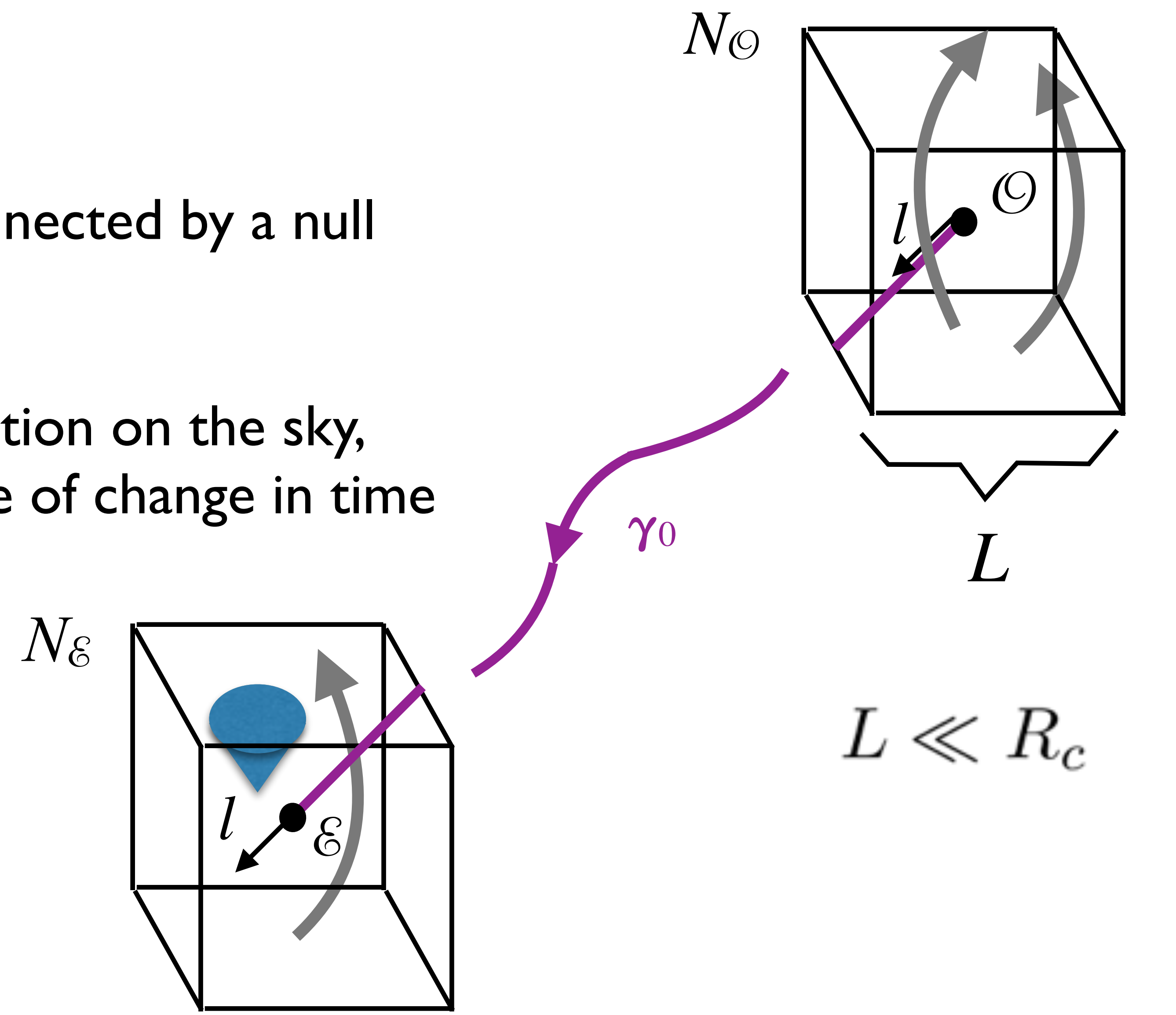

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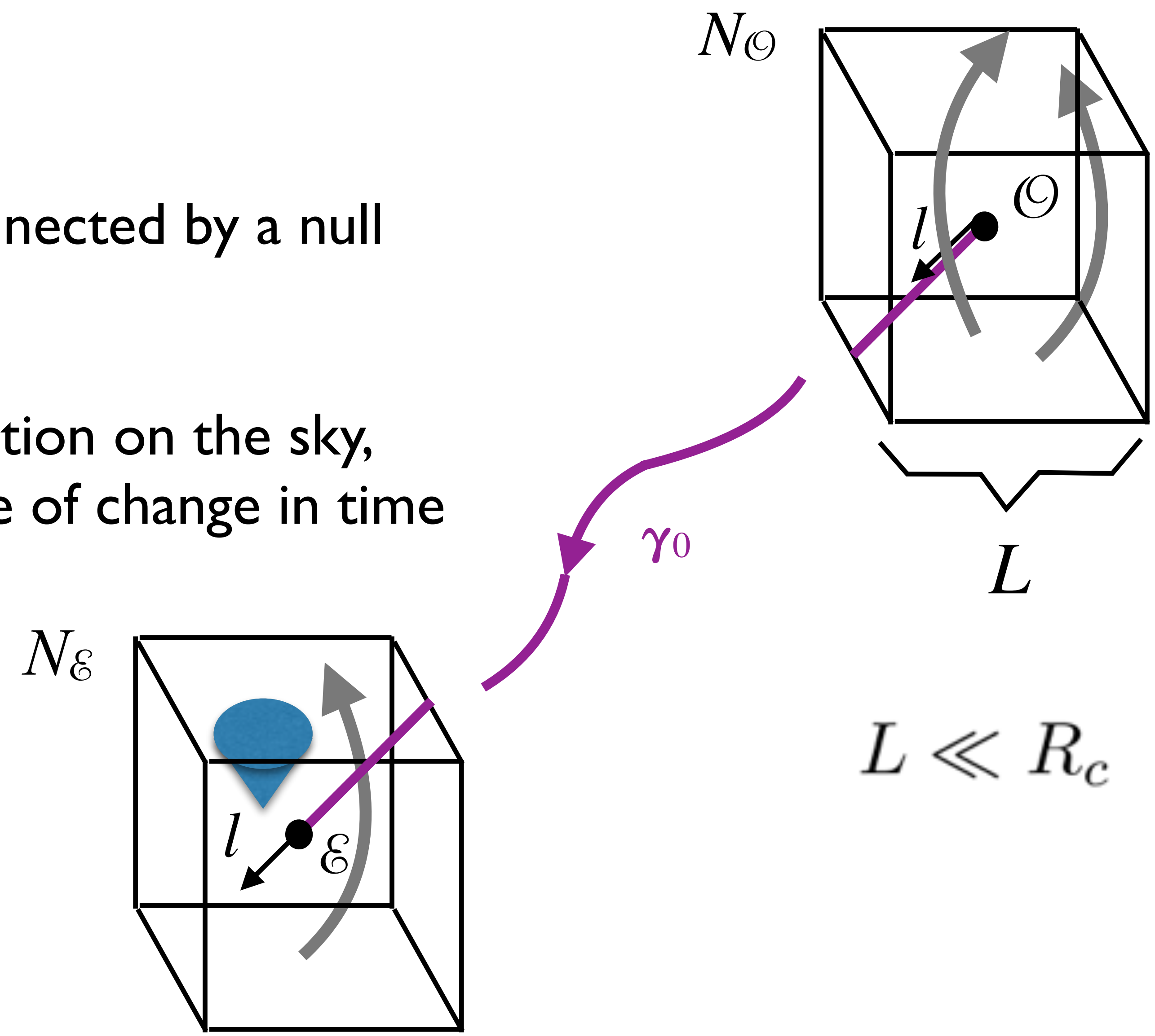

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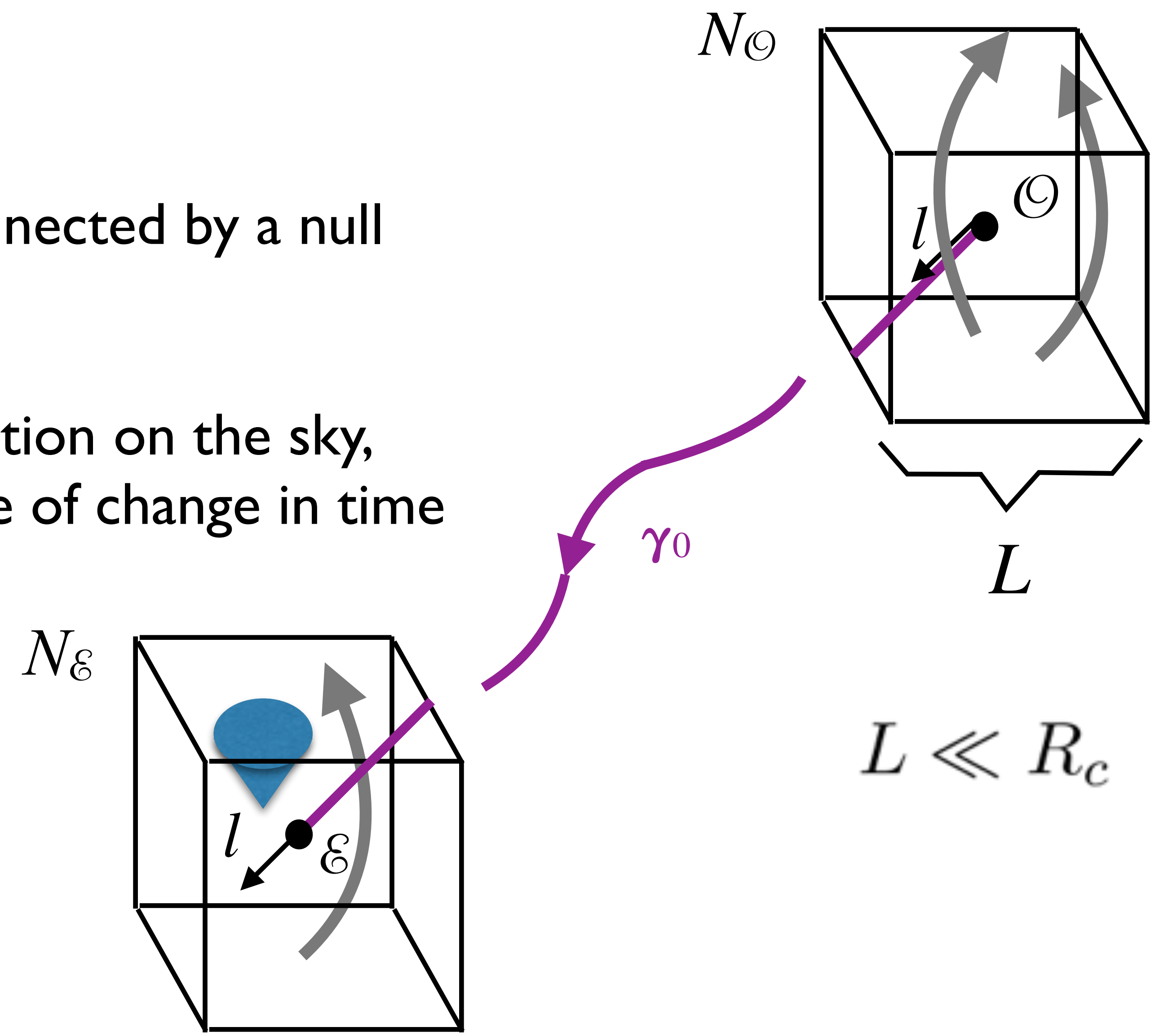

$$L \ll R_c$$

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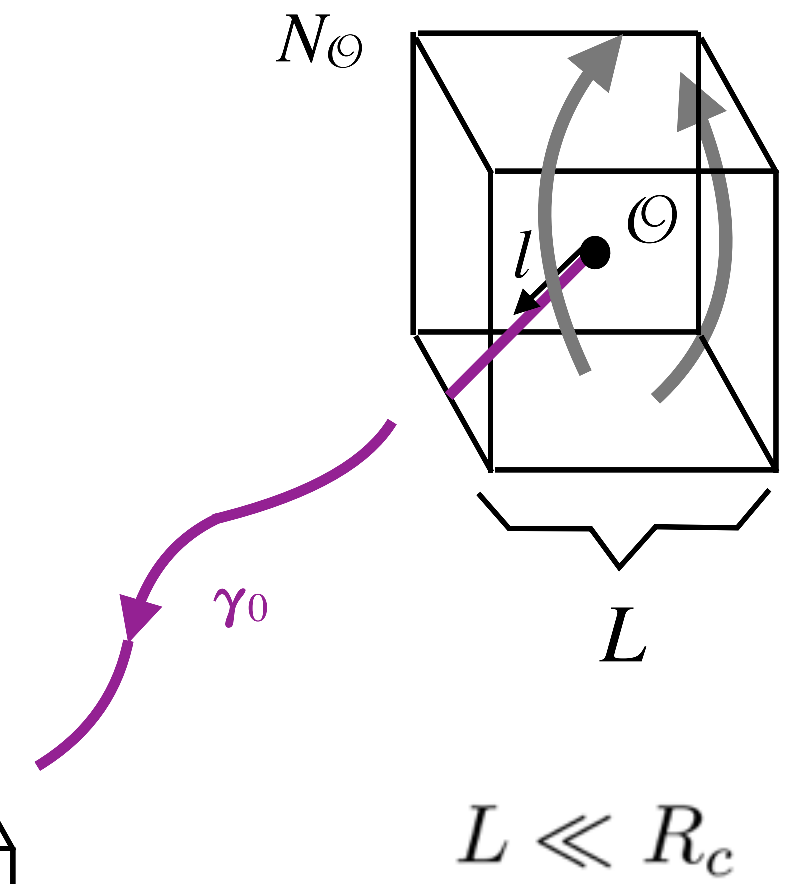
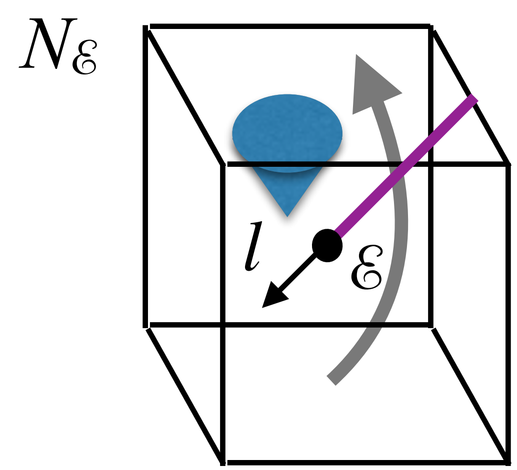
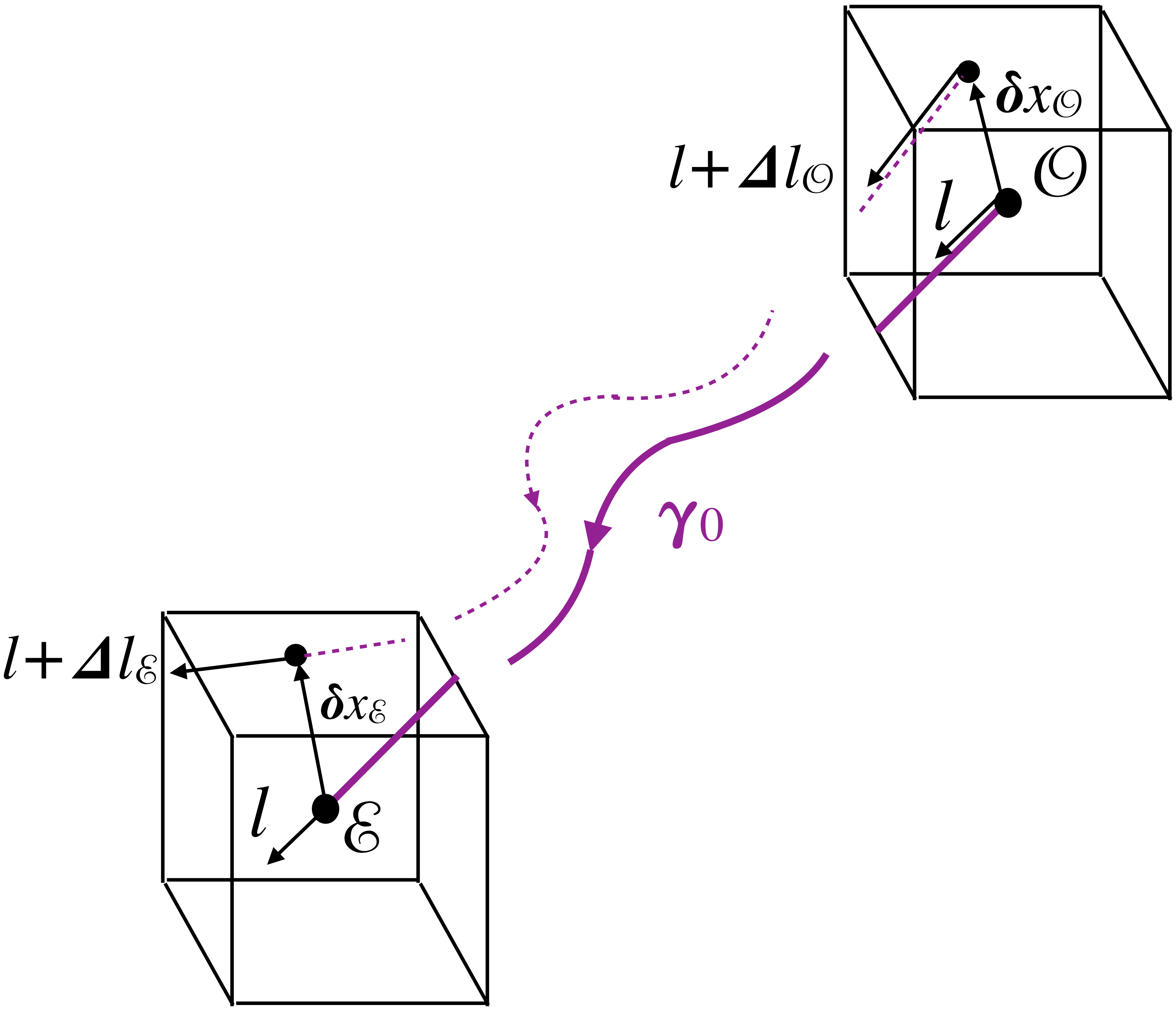




- Results affected by SR effects (time dilation, light aberration etc.)
- GR propagation effects through spacetime
    - Light bending, geometric time delay, Shapiro delay etc.
- Idea: geometric formulation of the problem (coordinate- and frame-independent)

Geometry

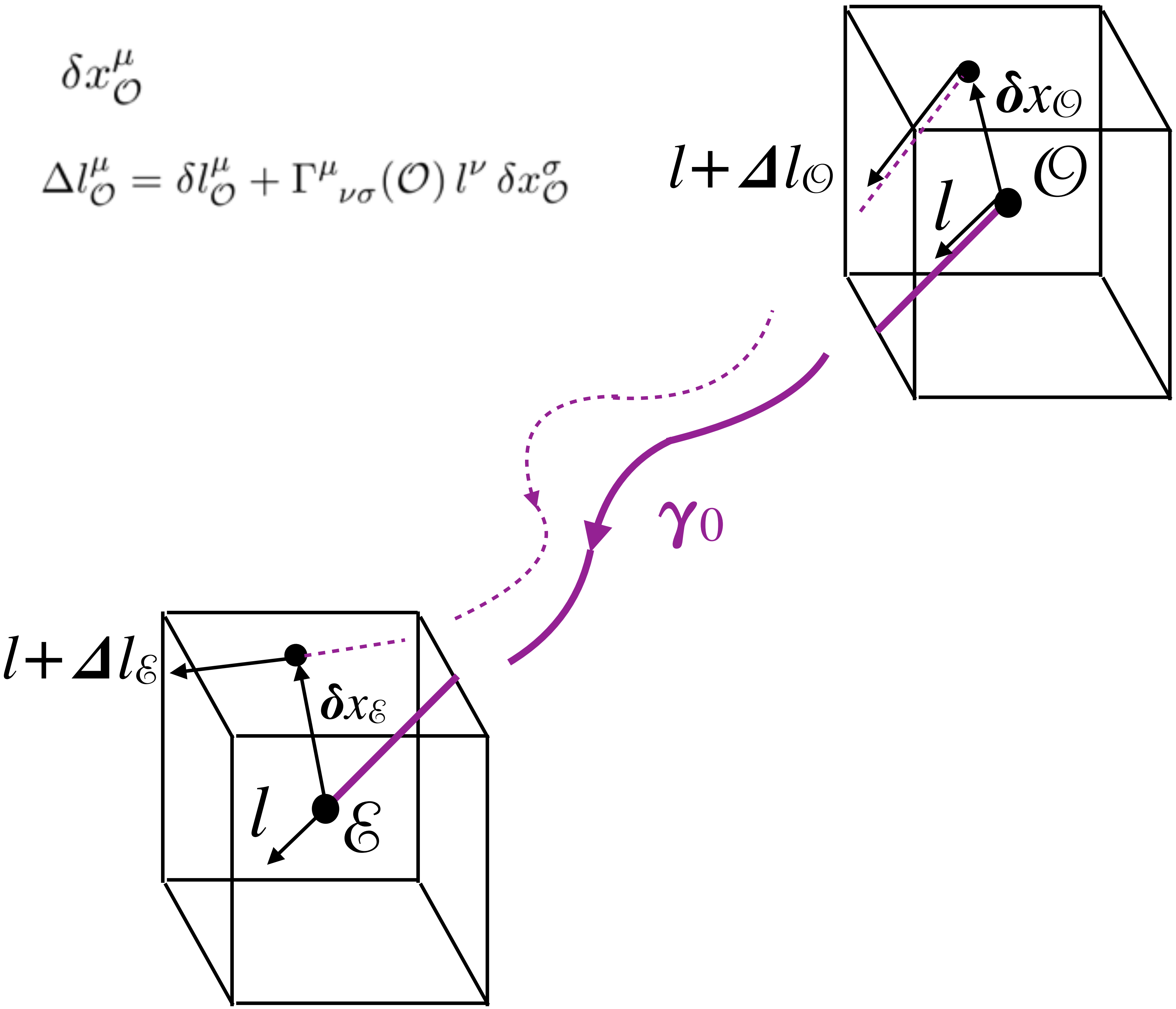


$\delta x_{\mathcal{O}}$
$l+\Delta l_{\mathcal{O}}$
$\mathcal{O}$
l
γ_0
$l+\Delta l_{\mathcal{E}}$
$\delta x_{\mathcal{E}}$
l
$\mathcal{E}$

Geometry

$\delta x^{\mu}_{\mathcal{O}}$

$\Delta l^{\mu}_{\mathcal{O}} = \delta l^{\mu}_{\mathcal{O}} + \Gamma^{\mu}{}_{\nu\sigma}(\mathcal{O})\, l^{\nu}\, \delta x^{\sigma}_{\mathcal{O}}$



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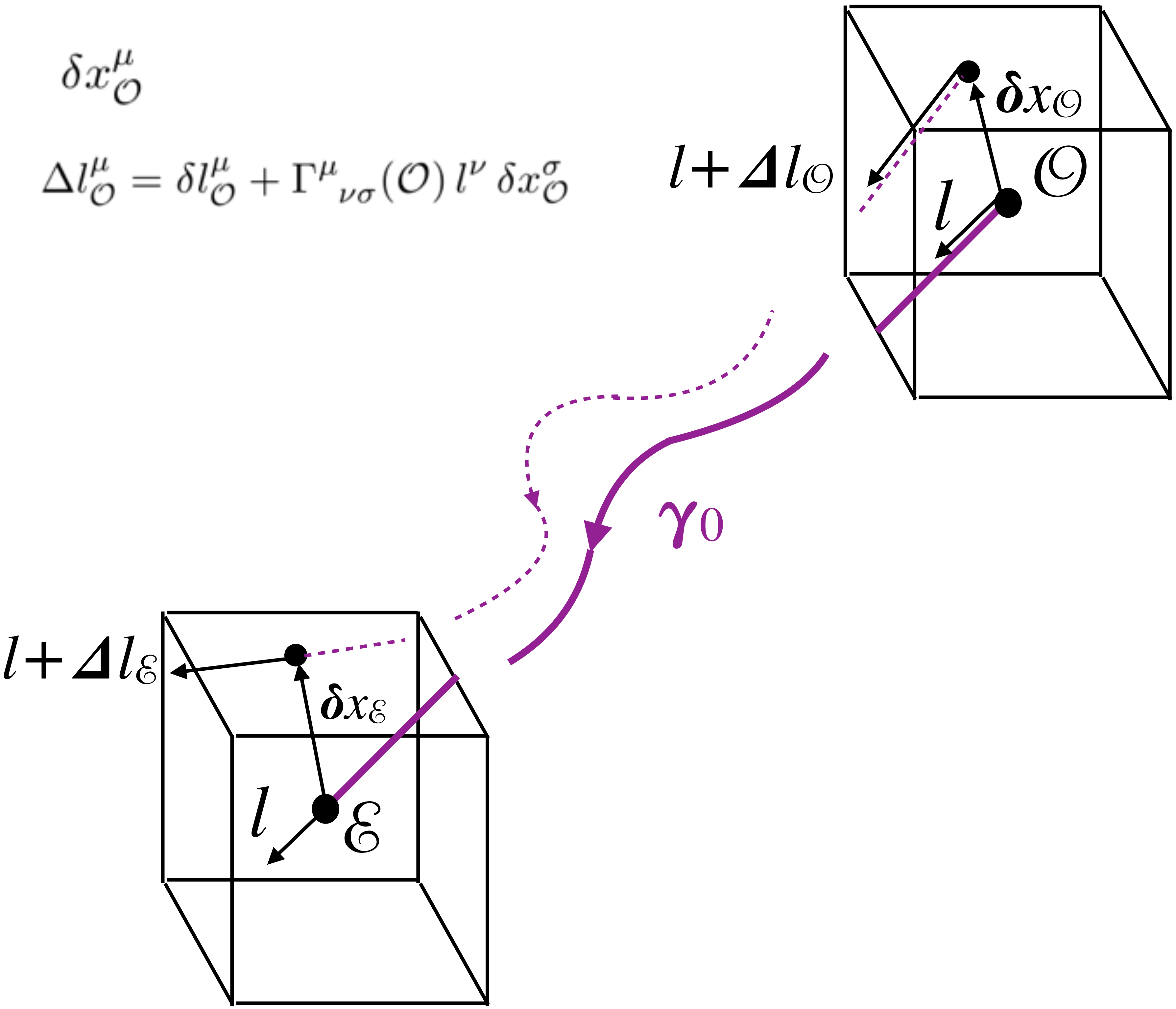
$\nabla_l \nabla_l \xi^\mu - R^\mu{}_{\alpha\beta\nu}\, l^\alpha\, l^\beta\, \xi^\nu = 0$

$\xi^\mu(\lambda_{\mathcal{O}}) = \delta x^\mu_{\mathcal{O}}$

$\nabla_l \xi^\mu(\lambda_{\mathcal{O}}) = \Delta l^\mu_{\mathcal{O}}$



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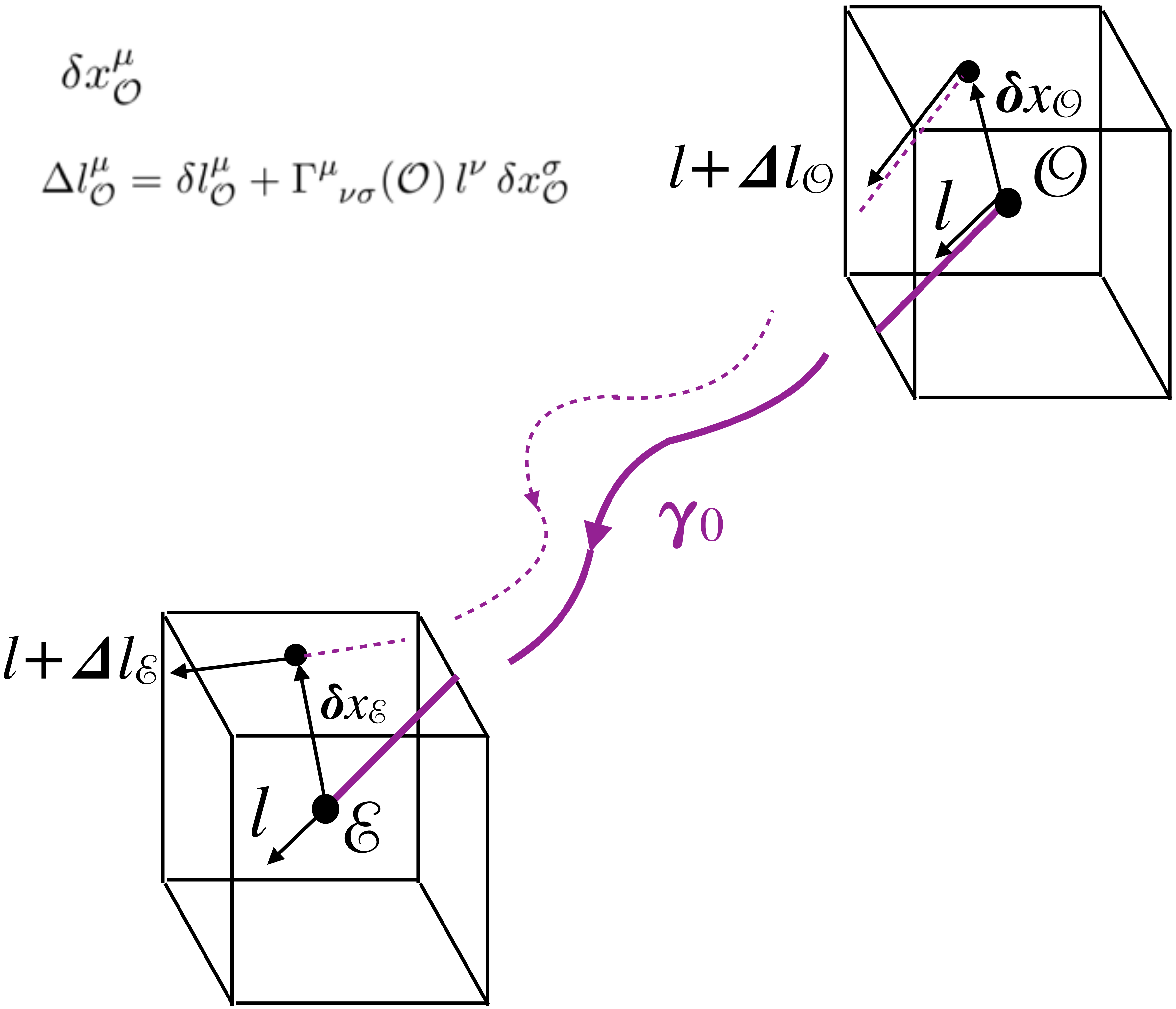

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$\delta x^\mu_{\mathcal{E}} = \xi^\mu(\lambda_{\mathcal{E}})$

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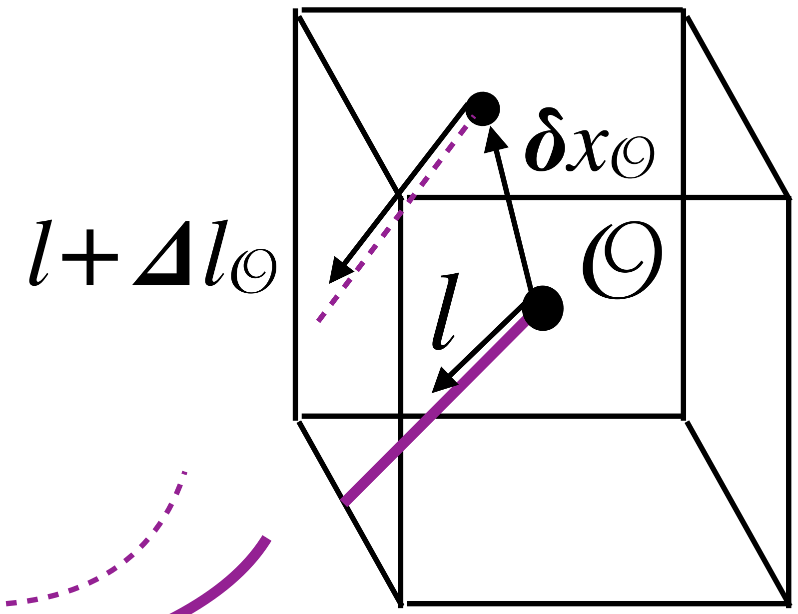

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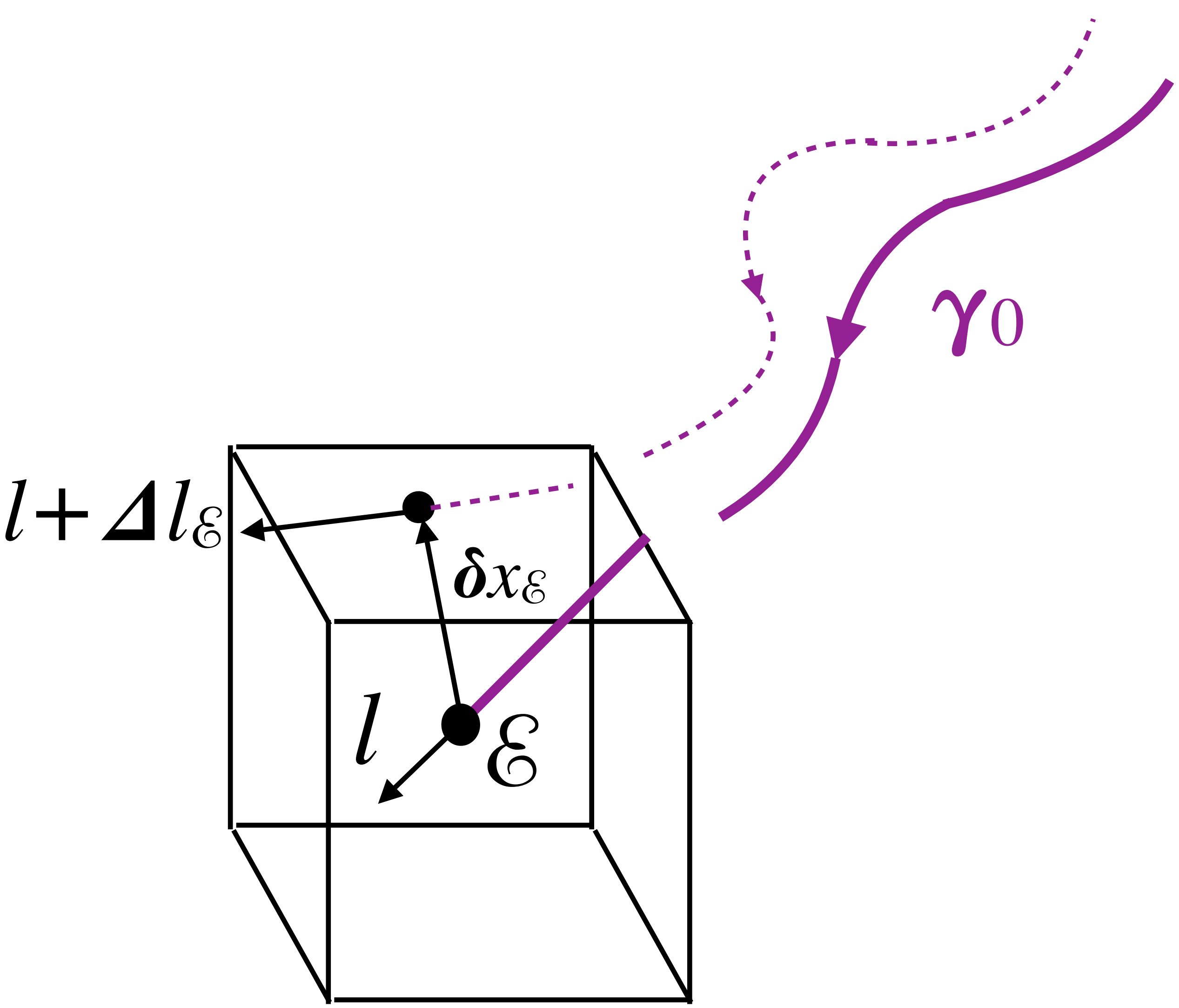

$$\delta x^\mu_{\mathcal{E}} = \xi^\mu(\lambda_{\mathcal{E}})$$
$$\Delta l^\mu_{\mathcal{E}} = \nabla_l \xi^\mu(\lambda_{\mathcal{E}})$$

$$\begin{aligned}\delta x^\mu_{\mathcal{E}} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \\ \Delta l^\mu_{\mathcal{E}} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}}\end{aligned}$$

Geometry

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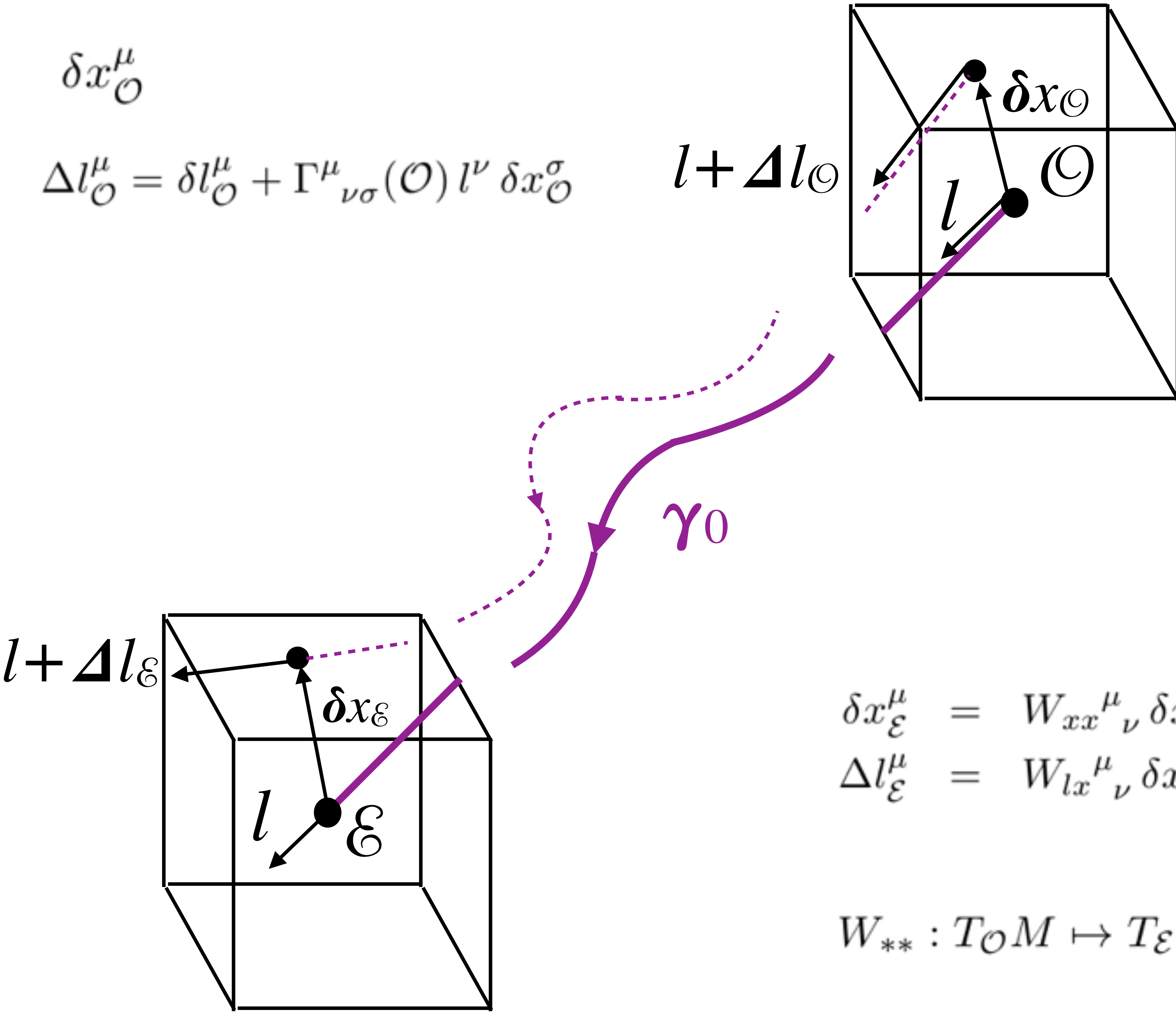

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$$W_{**} : T_\mathcal{O} M \mapsto T_\mathcal{E} M$$

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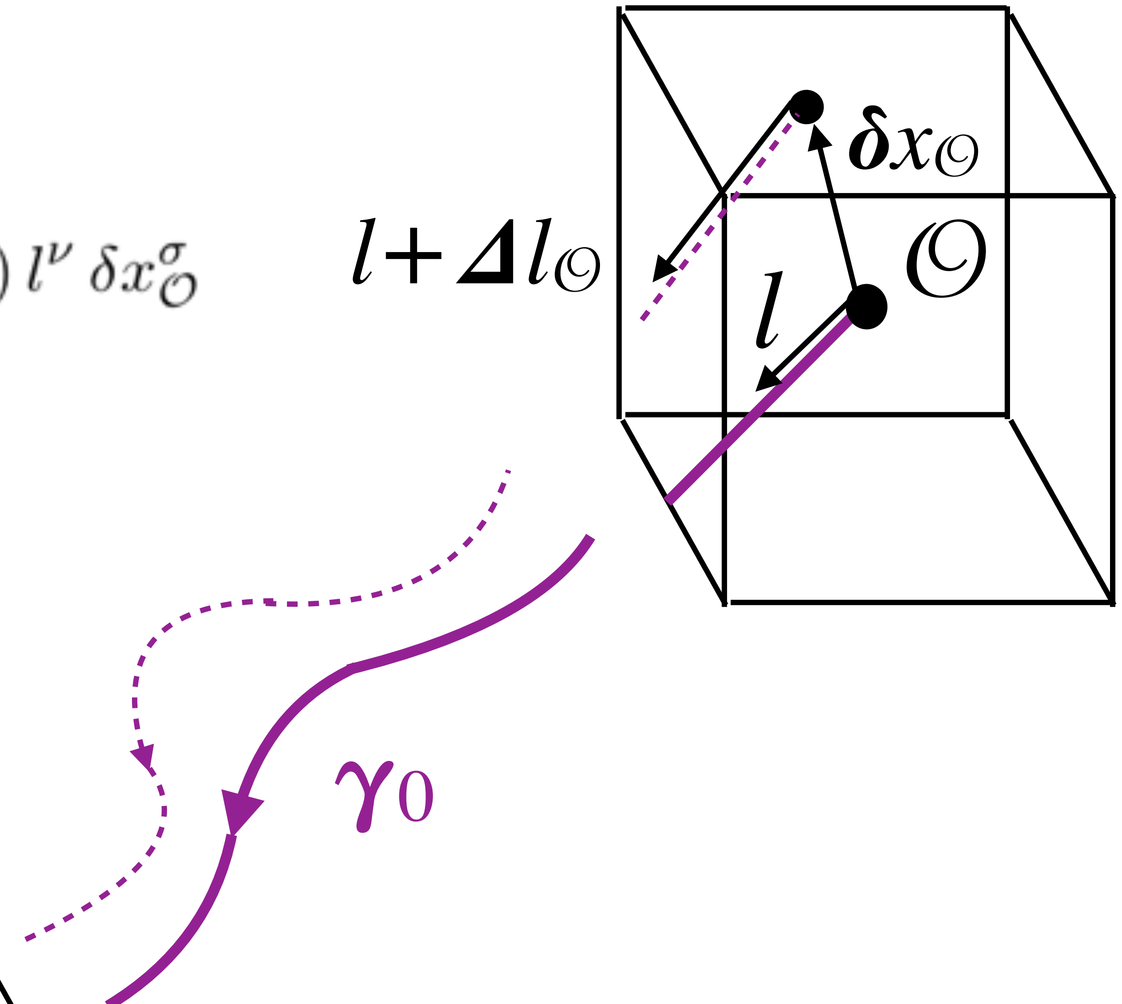

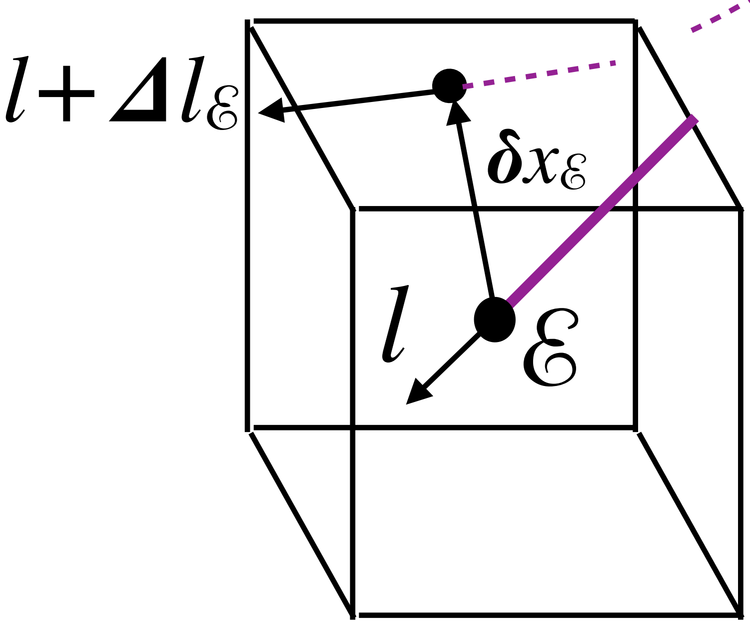

$$\ddot{A}^\mu{}_\nu - R^\mu{}_{\alpha\beta\sigma}\, l^\alpha\, l^\beta\, A^\sigma{}_\nu = 0$$
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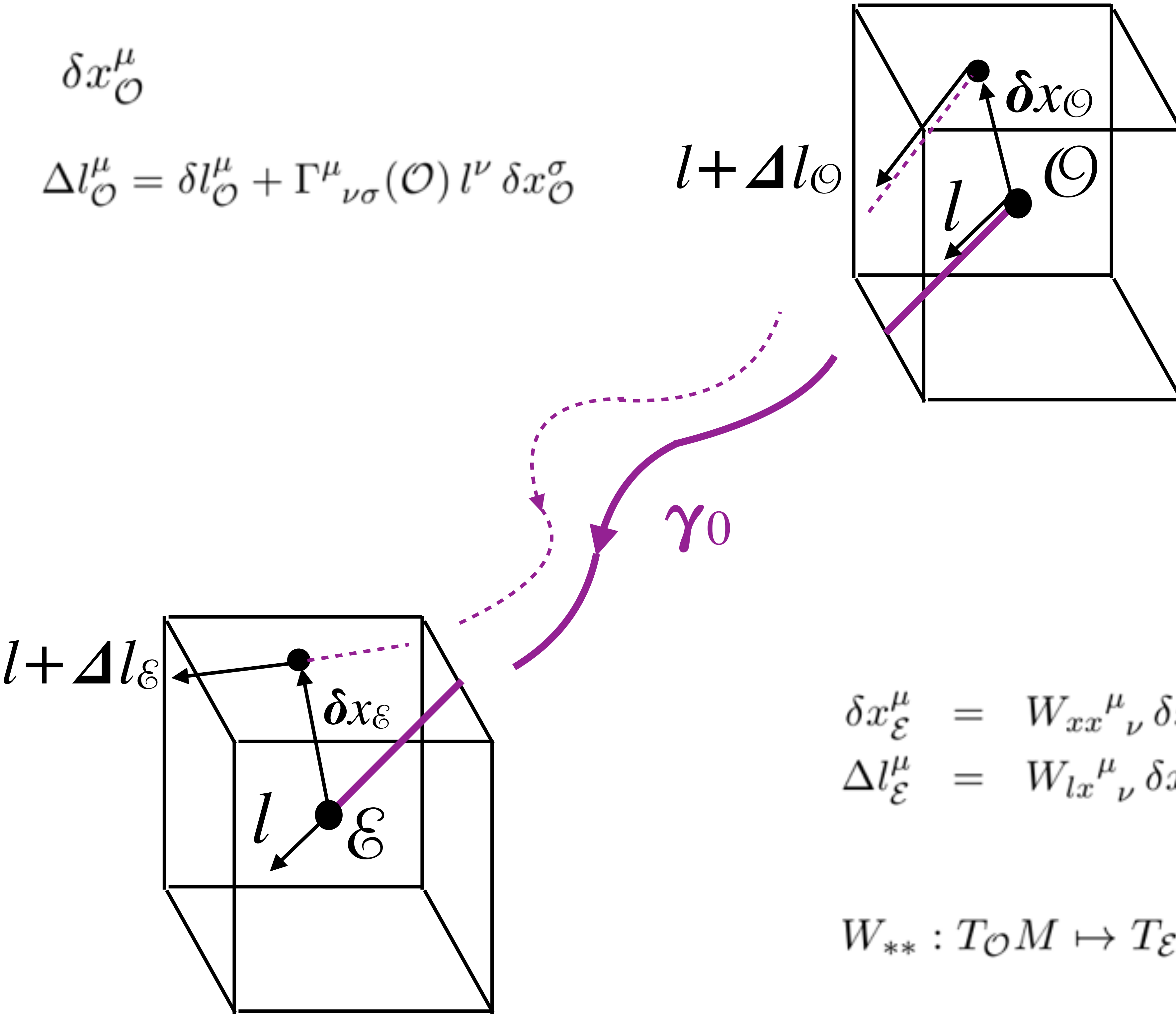

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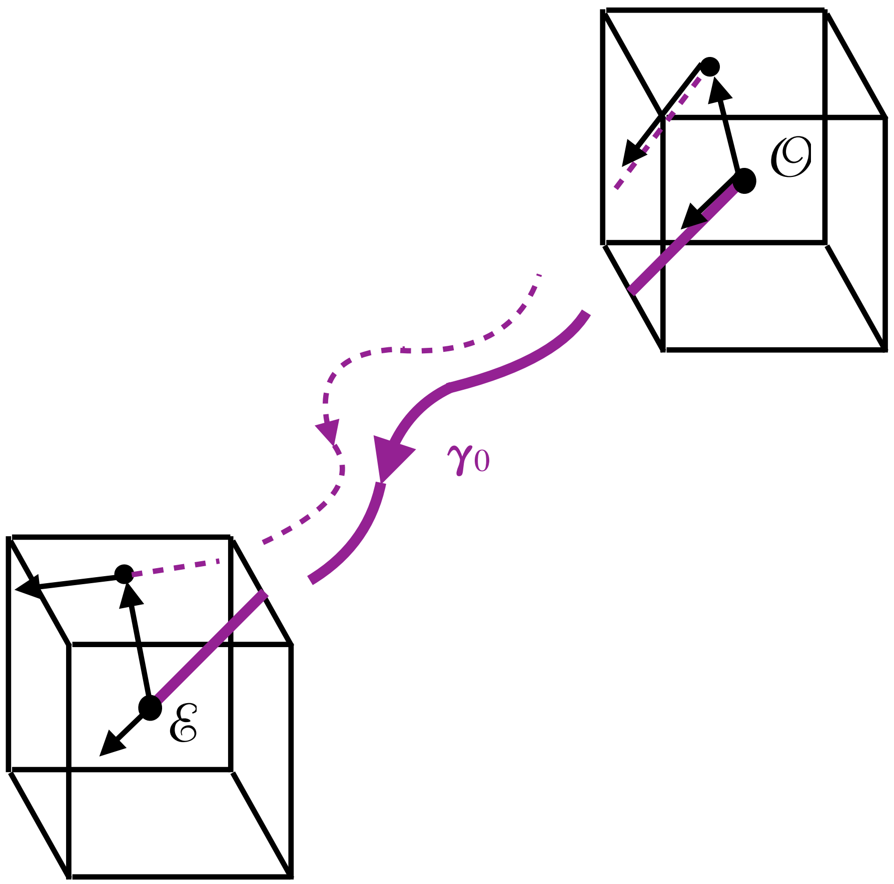
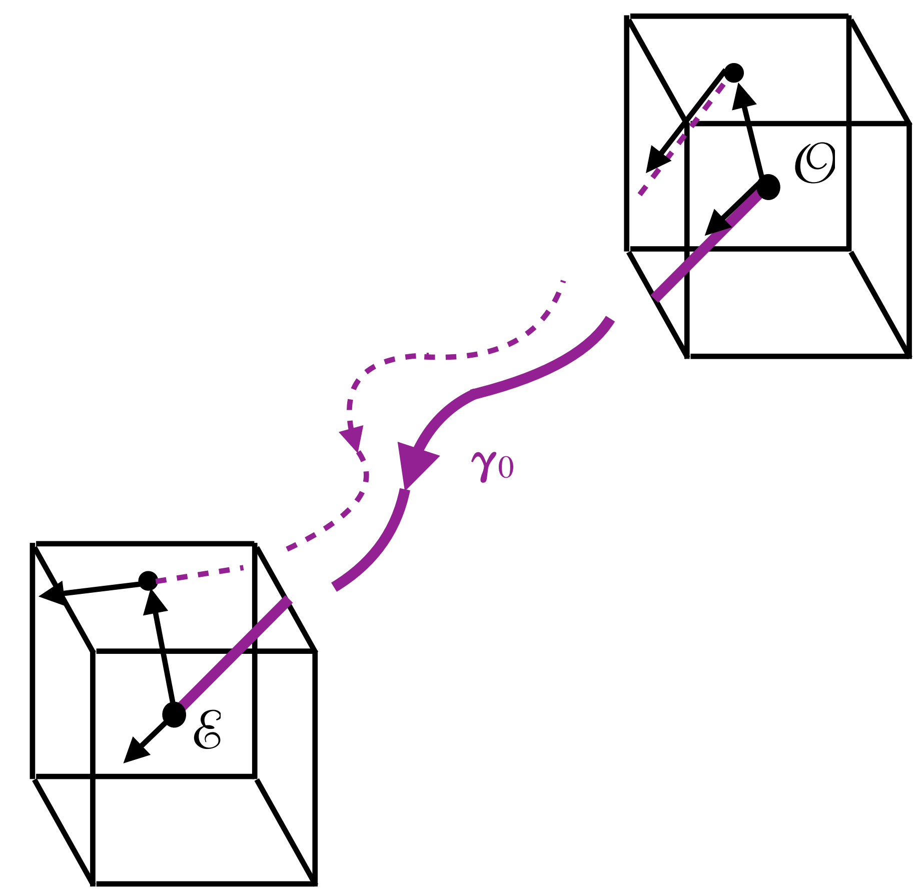


Geometry

- W_{**} contain information about *all* geodesics from $N_{\mathcal{O}}$ to $N_{\mathcal{E}}$



Geometry

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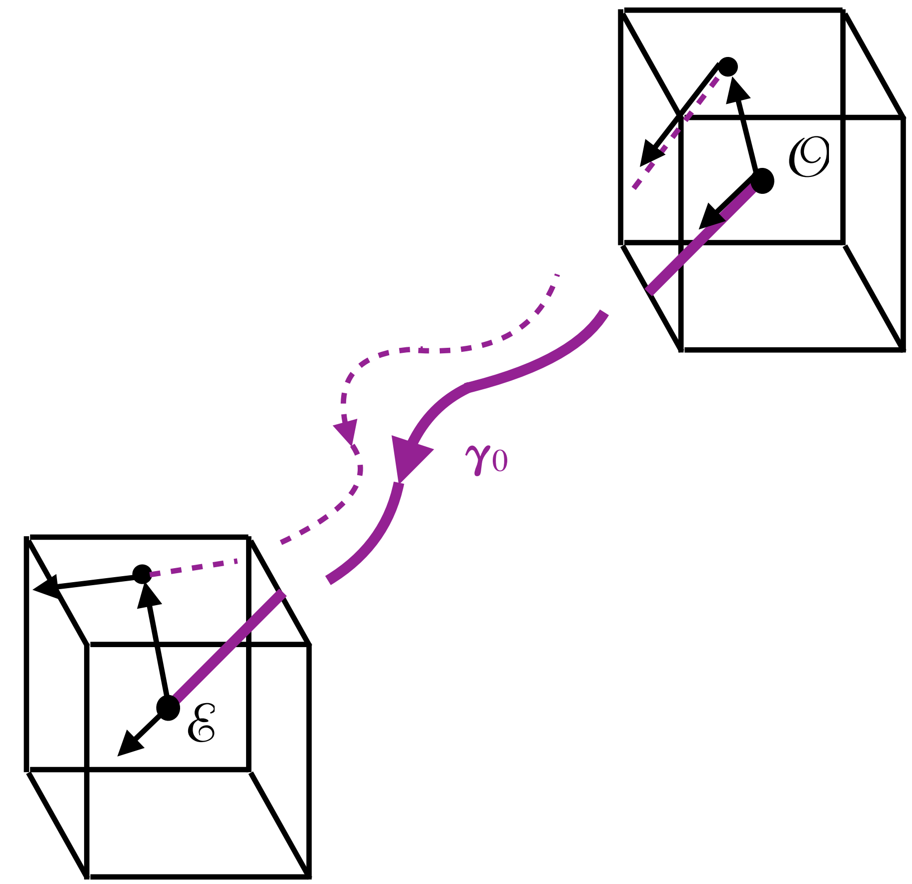

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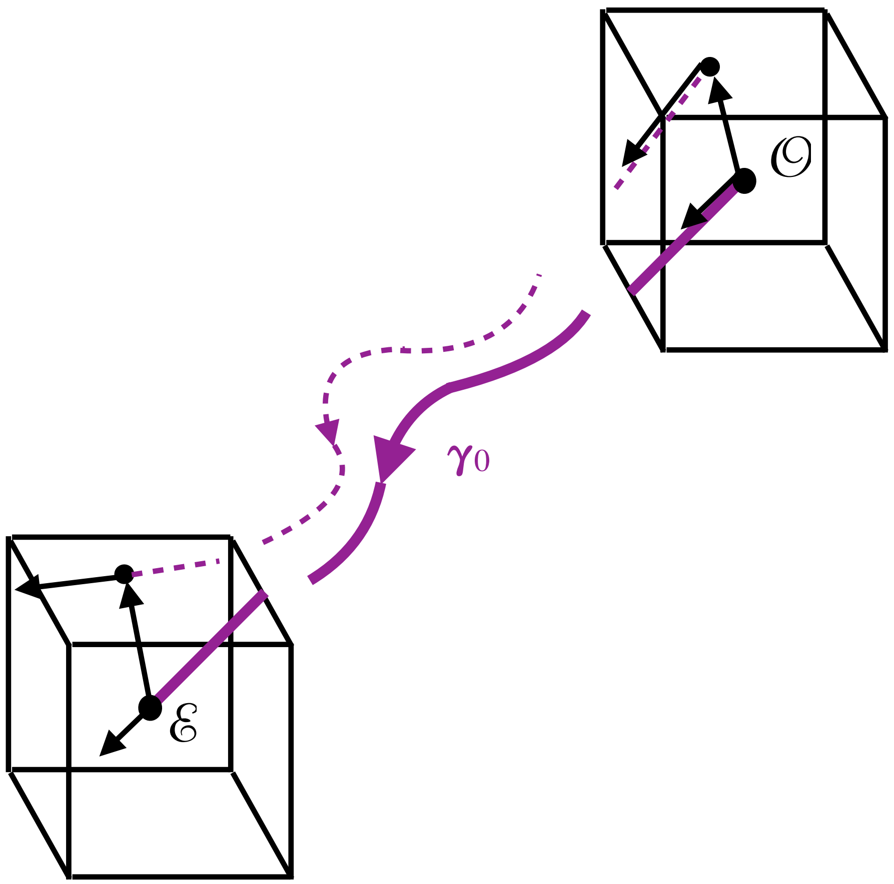

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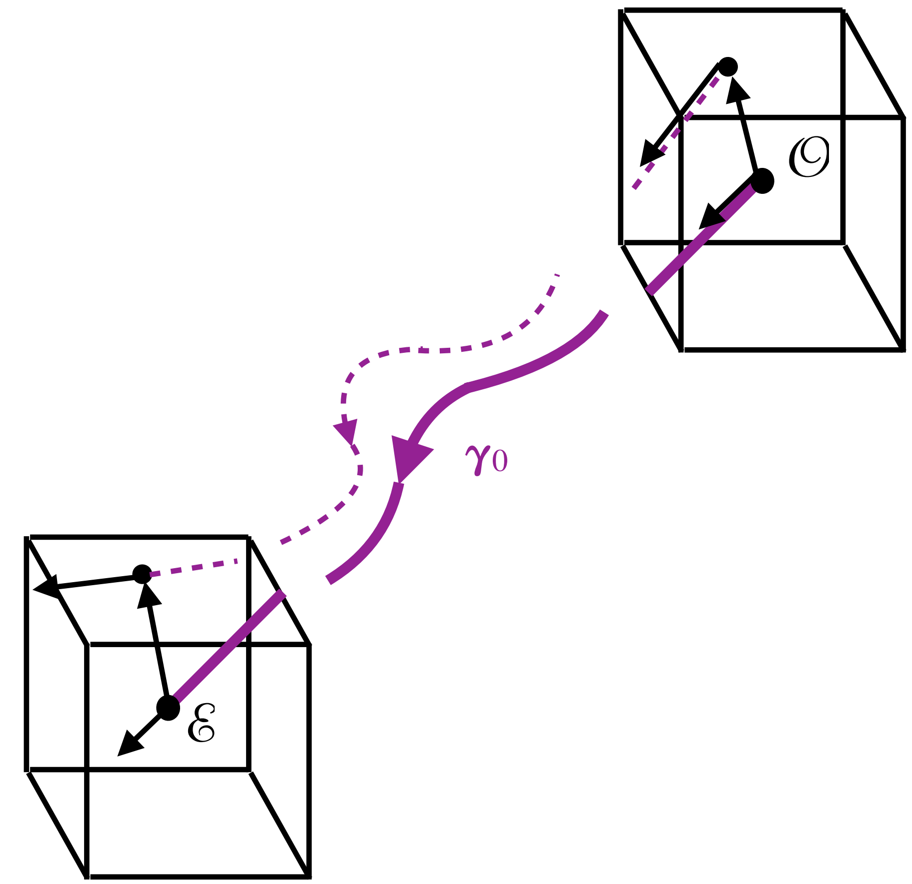

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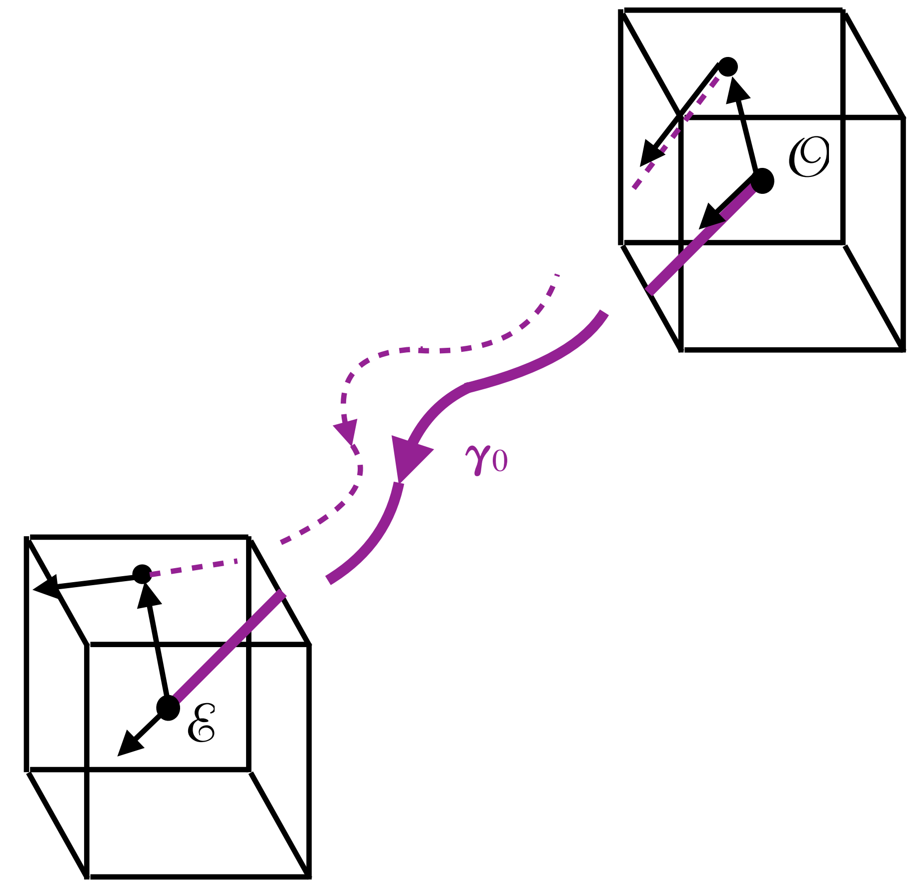
  - geometric optics - only the geodesic's path matters. We can identify null geodesics differing by affine reparametrization



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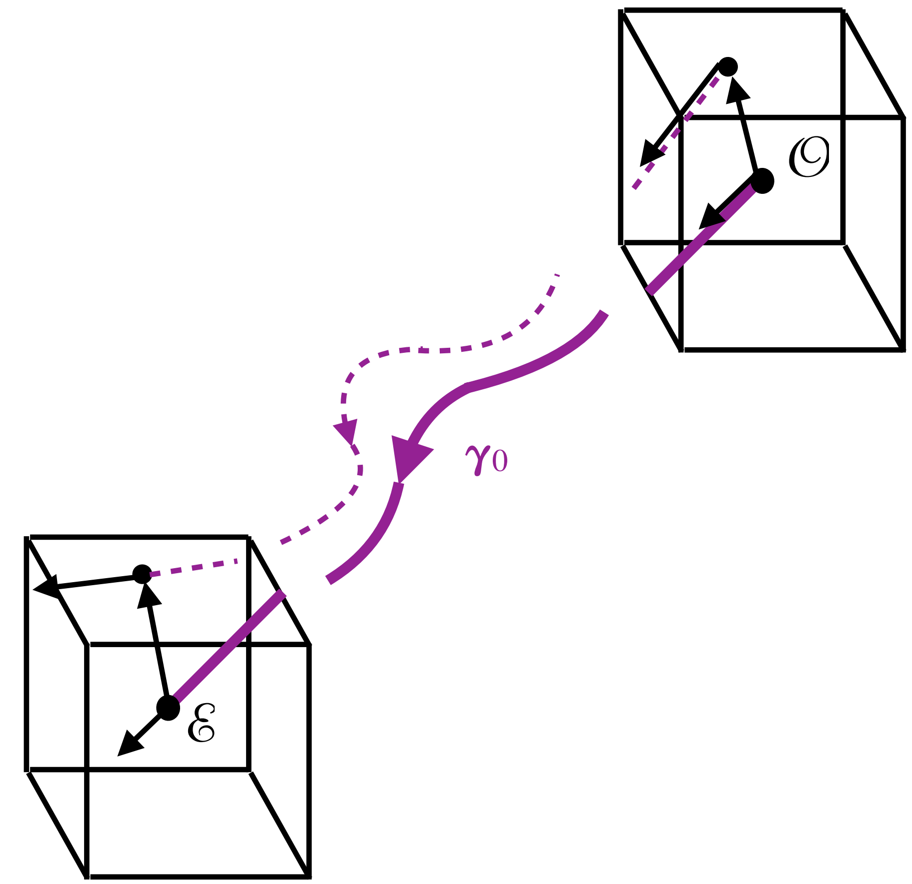

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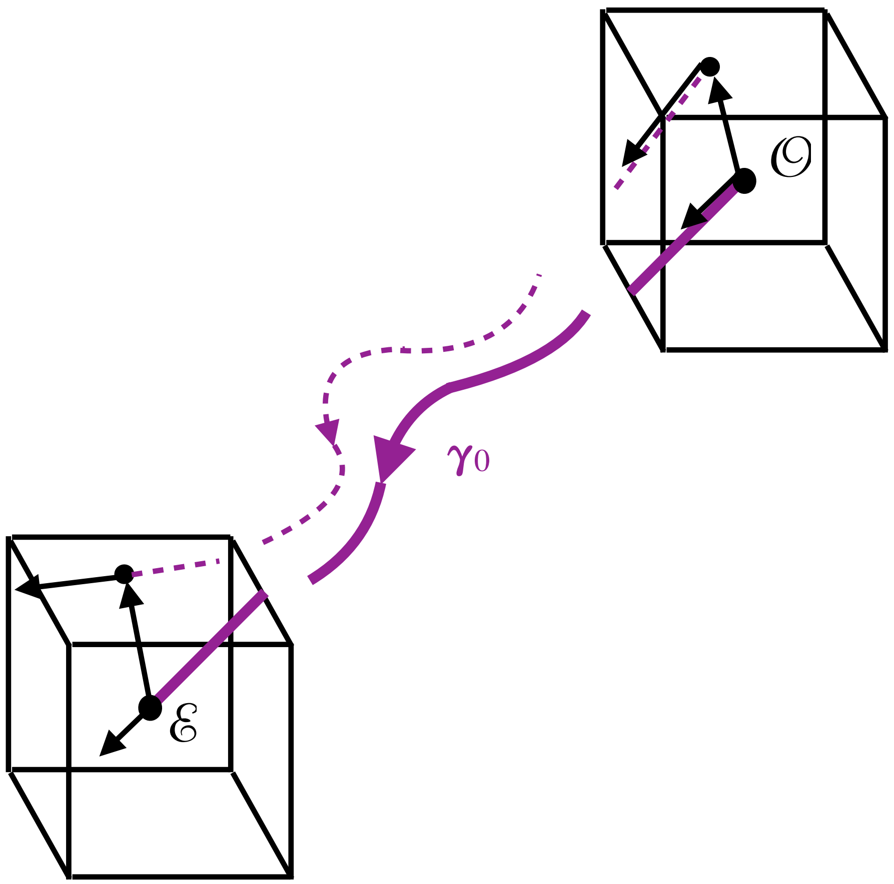

Dim = 8 - 1 - 2 = 5

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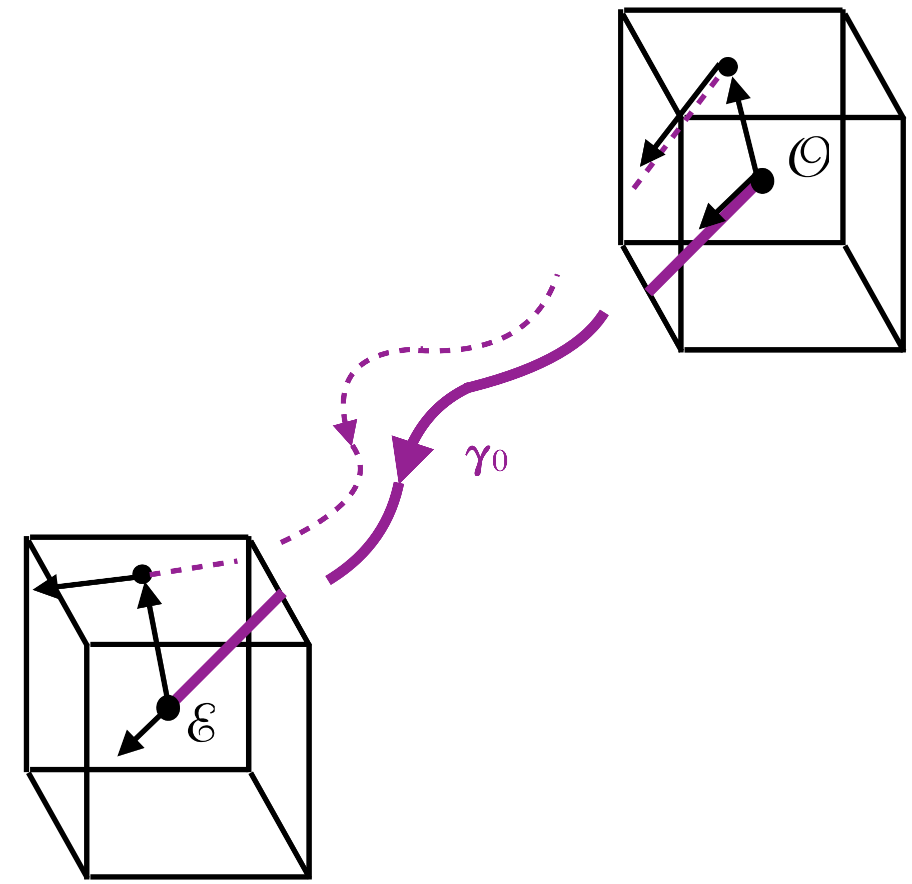

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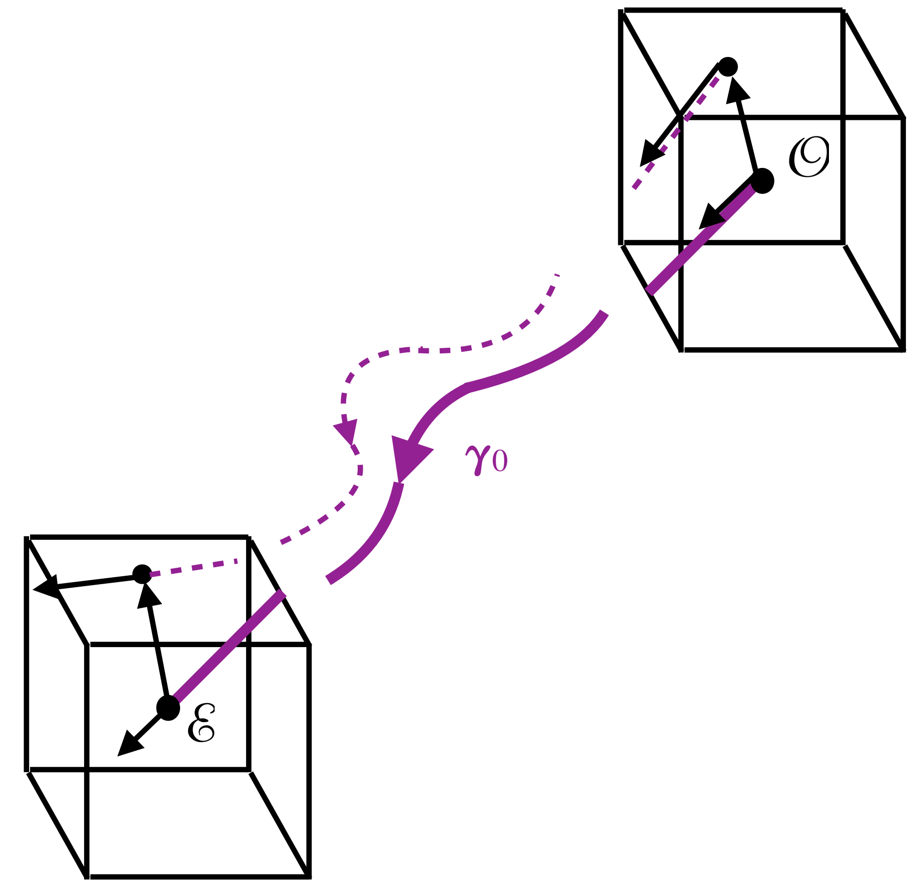

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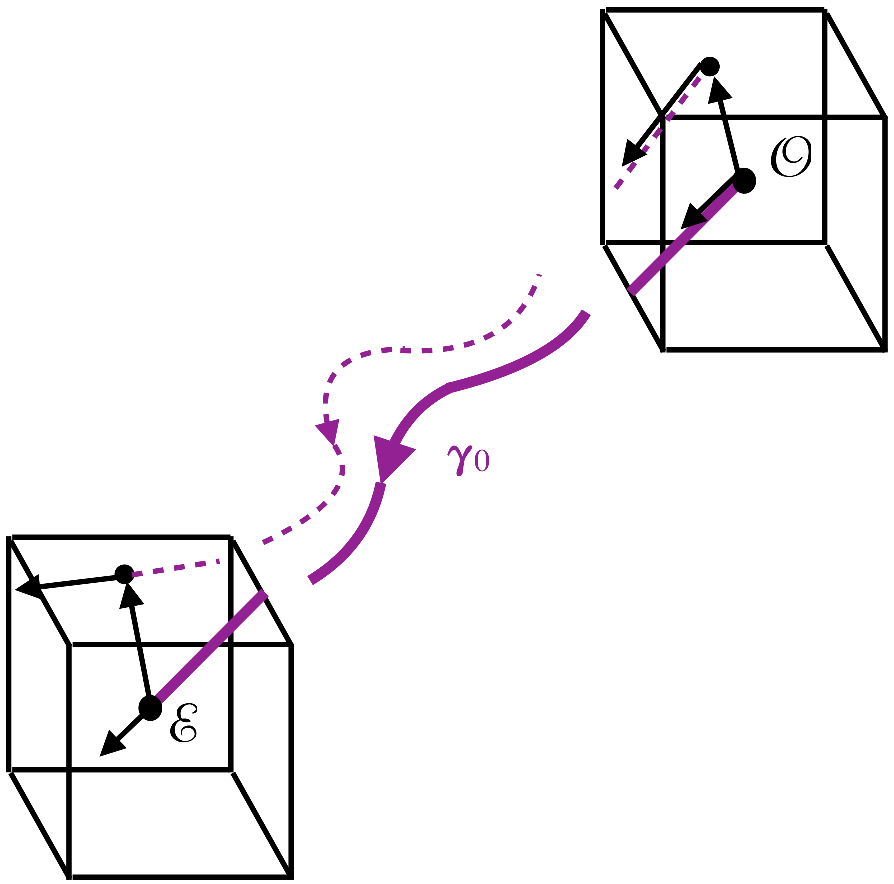

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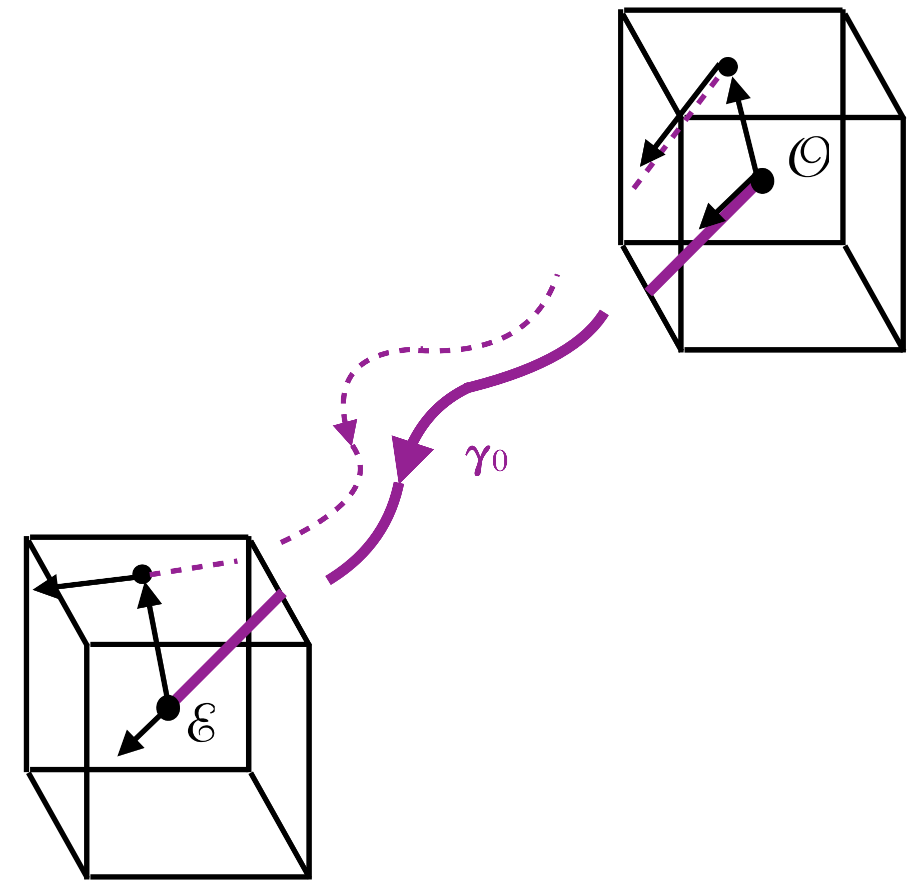

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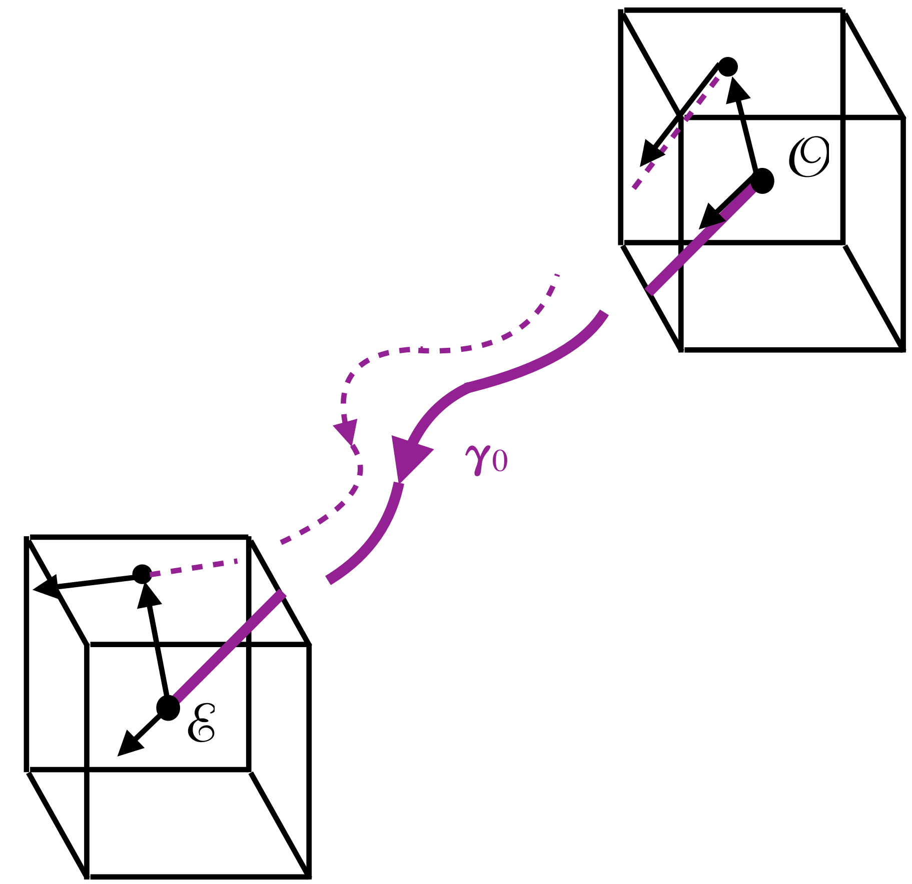
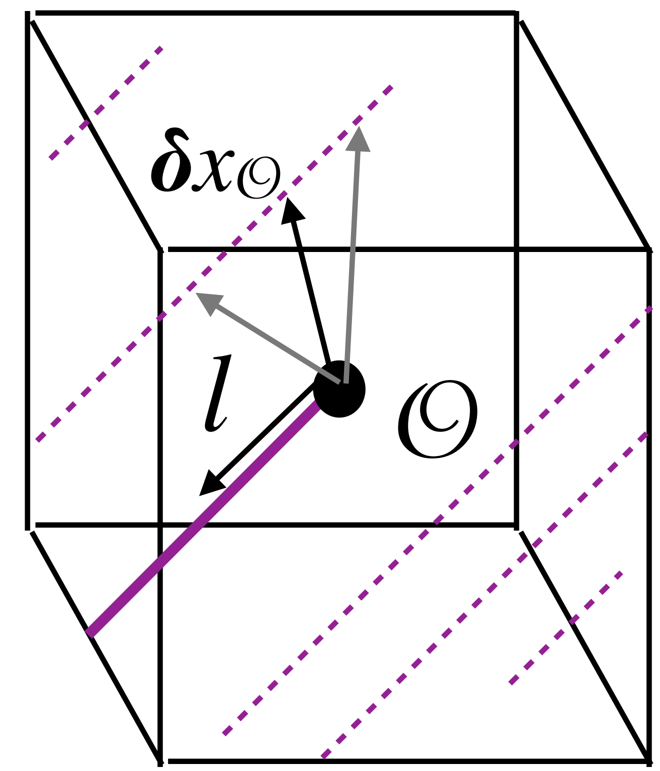


Dim = 8 - 1 - 2 = 5

$$\begin{pmatrix} \delta x^\mu_{\mathcal{O}} \\ \Delta l^\mu_{\mathcal{O}} \end{pmatrix} \sim \begin{pmatrix} \delta x^\mu_{\mathcal{O}} + \delta C_1\, l^\mu \\ \Delta l^\mu_{\mathcal{O}} + \delta C_2\, l^\mu \end{pmatrix} \Longleftrightarrow \begin{pmatrix} \delta x^\mu_{\mathcal{E}} + \delta C_3\, l^\mu \\ \Delta l^\mu_{\mathcal{E}} + \delta C_2\, l^\mu \end{pmatrix} \sim \begin{pmatrix} \delta x^\mu_{\mathcal{E}} \\ \Delta l^\mu_{\mathcal{E}} \end{pmatrix}$$

- Distant observer approximation (DOA) $\quad \Delta l^\mu_{\mathcal{O}} \ll l^\mu_{\mathcal{O}}$

Geometry

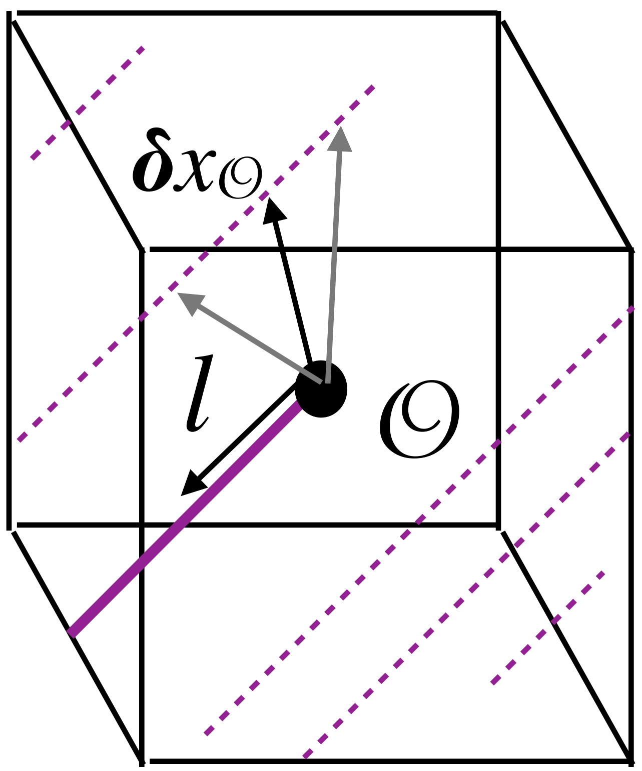


Geometry

$[\boldsymbol{\delta} x_{\mathcal{O}}] \in \mathrm{T}_{\mathcal{O}}M \,/\, l$

$[\boldsymbol{\delta} x_{\mathcal{O}}] = \{\, X^{\mu} \in \mathrm{T}_{\mathcal{O}}M \mid X^{\mu} = \boldsymbol{\delta} x_{\mathcal{O}}{}^{\mu} + C\, l^{\mu} \,\}$

$\dim\,(\mathrm{T}_{\mathcal{O}}M \,/\, l) = 3$



Geometry

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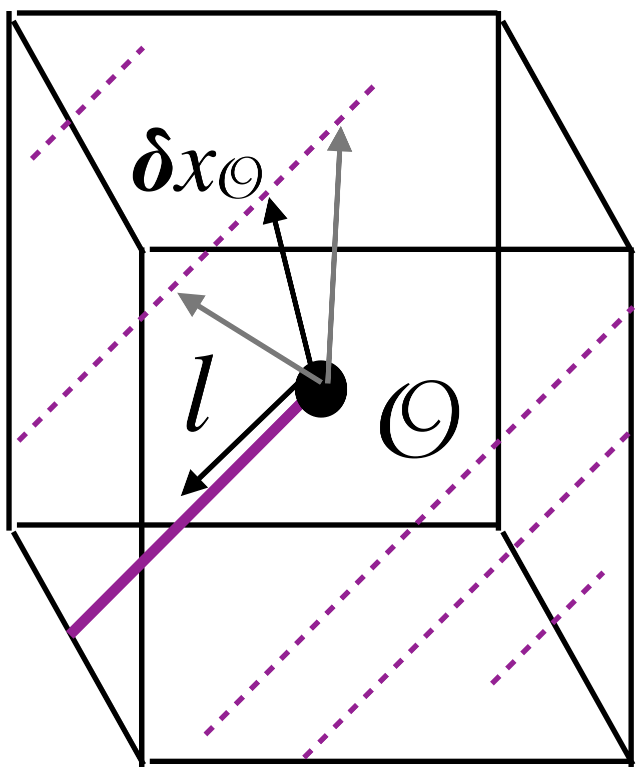
$[\boldsymbol{\Delta} l_{\mathcal{O}}] \in l^{\perp} / l = \mathcal{P}_{\mathcal{O}}$

$[\boldsymbol{\Delta} l_{\mathcal{O}}] = \{\, X^{\mu} \in l^{\perp} \mid X^{\mu} = \boldsymbol{\Delta} l_{\mathcal{O}}{}^{\mu} + \mathrm{C}\, l^{\mu} \,\}$

$\dim \mathcal{P}_{\mathcal{O}} = 2$



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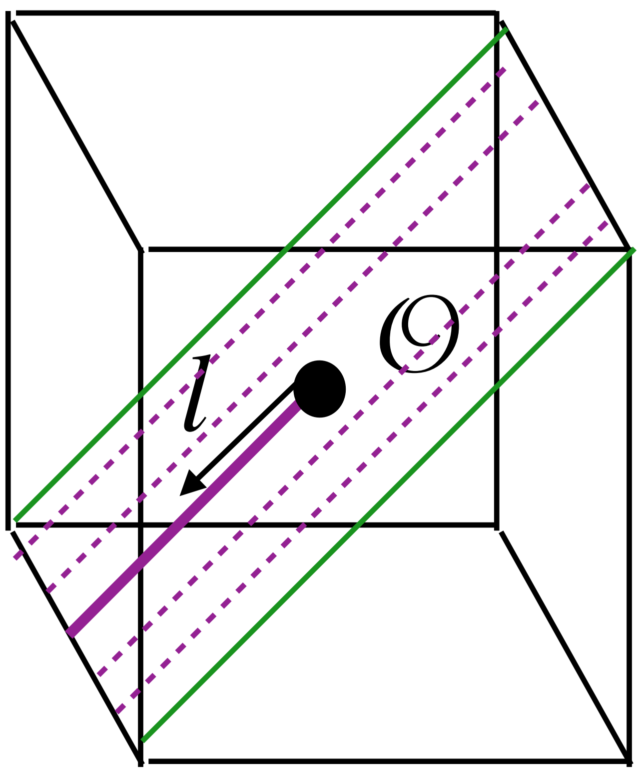

Geometry

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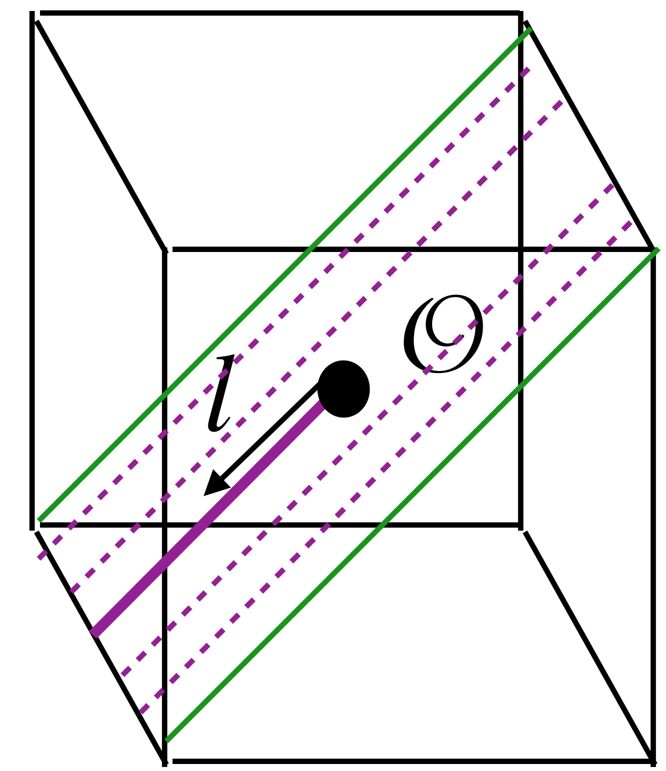

- Perpendicular space - corresponds to direction deviations of null geodesics

Geometry

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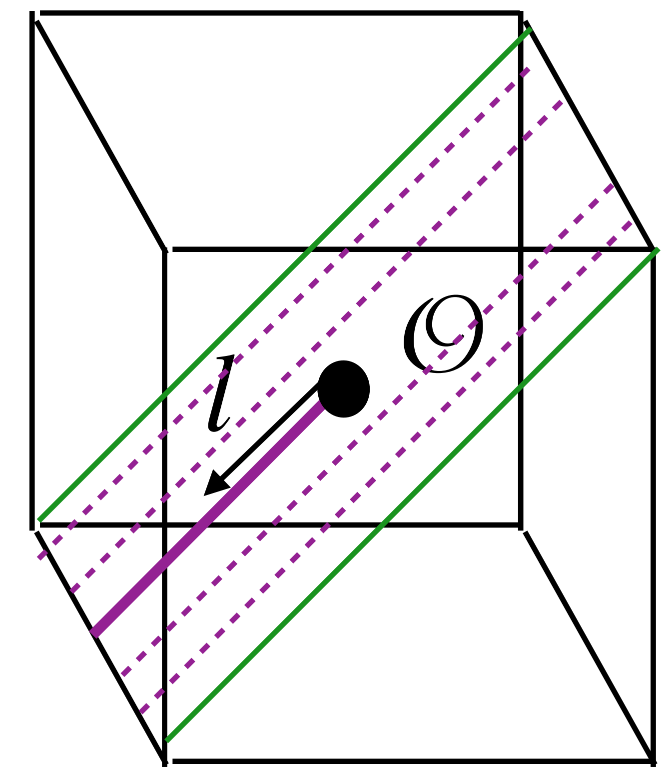

- Perpendicular space - corresponds to direction deviations of null geodesics
- …or to displacement of null geodesics on the same light front

Geometry

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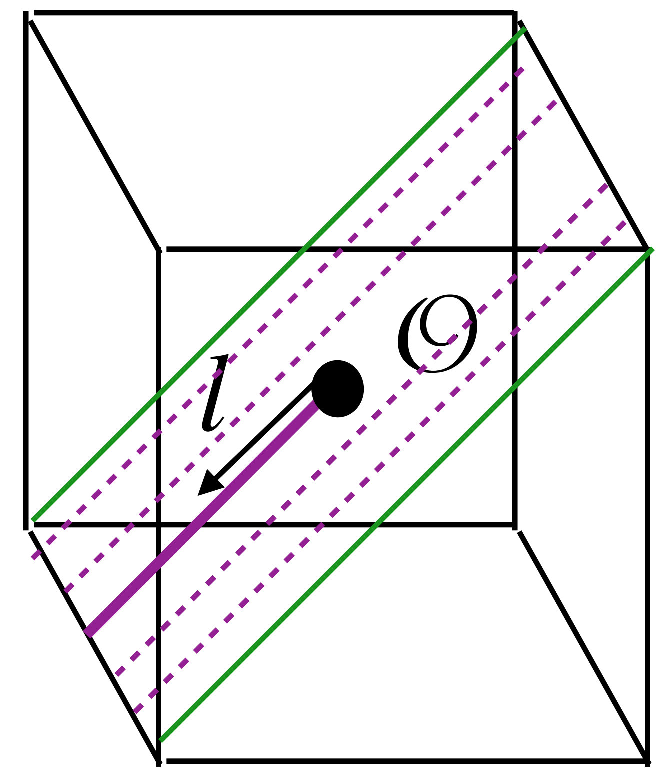

- Perpendicular space - corresponds to direction deviations of null geodesics
- …or to displacement of null geodesics on the same light front
- inherits a positive definite metric $[g]$ from the spacetime metric

Geometry

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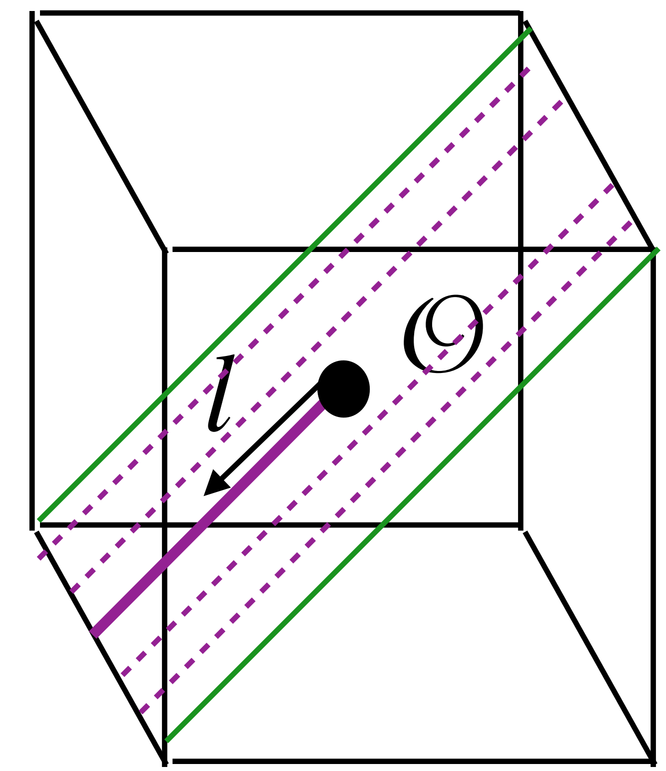


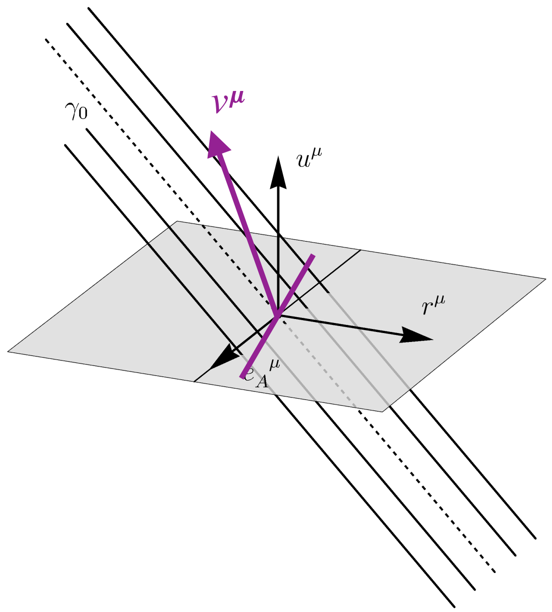
- Perpendicular space - corresponds to direction deviations of null geodesics
- …or to displacement of null geodesics on the same light front
- inherits a positive definite metric $[g]$ from the spacetime metric
- can be identified with the screen space of any observer

Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

$$\delta x^\mu_{\mathcal{O}}\, l_\mu = 0$$



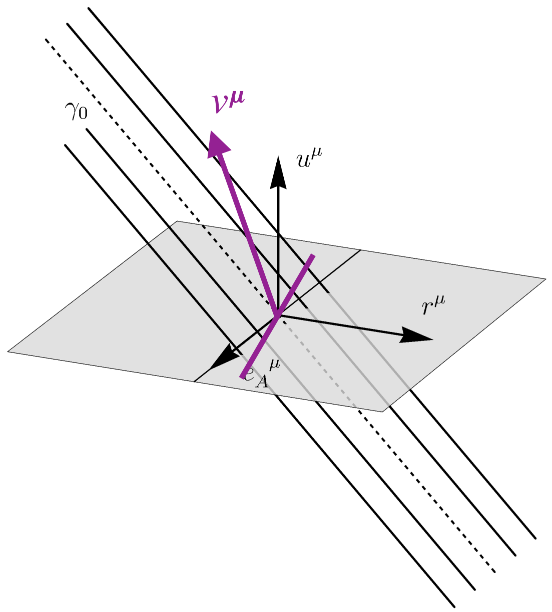
Geometry

Perpendicular space $\mathscr{P}$

Different observers u and v ⇒ different notions of simultaneity

- orthogonally displaced null geodesics

$$\delta x_{\mathcal{O}}^{\mu}\, l_{\mu} = 0$$



Geometry

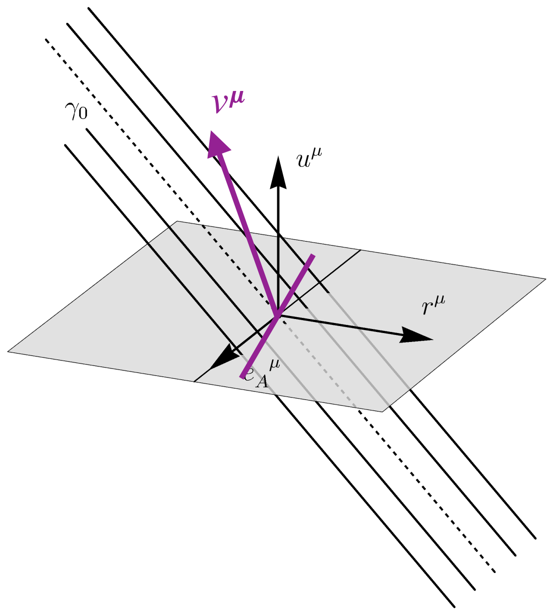
Perpendicular space $\mathscr{P}$

Different observers u and v ⇒ different notions of simultaneity

⇒ different screen spaces

- orthogonally displaced null geodesics

$$\delta x^{\mu}_{\mathcal{O}}\, l_{\mu} = 0$$



Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

Different observers u and v ⇒ different notions of simultaneity

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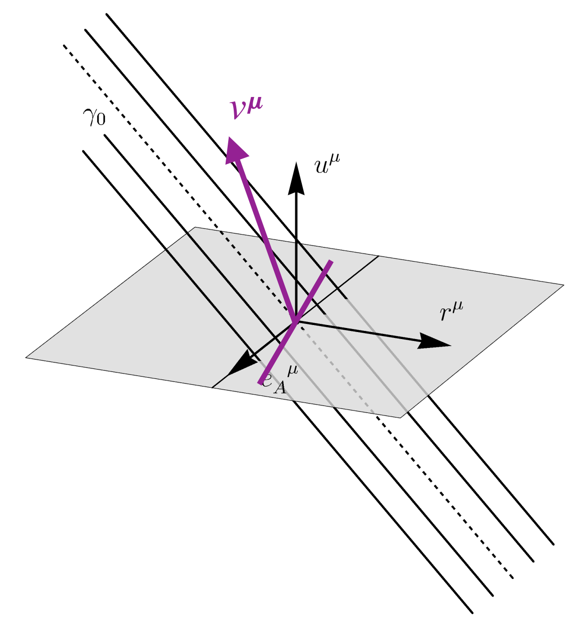


⇒ different screen spaces

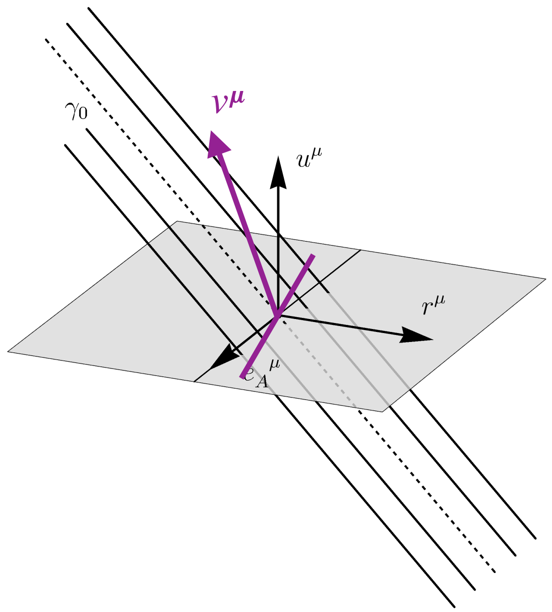
⇒ screen spaces punctured by light rays at different points

Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

$$\delta x^\mu_{\mathcal{O}}\, l_\mu = 0$$



Different observers u and v ⇒ different notions of simultaneity

⇒ different screen spaces

⇒ screen spaces punctured by light rays at different points

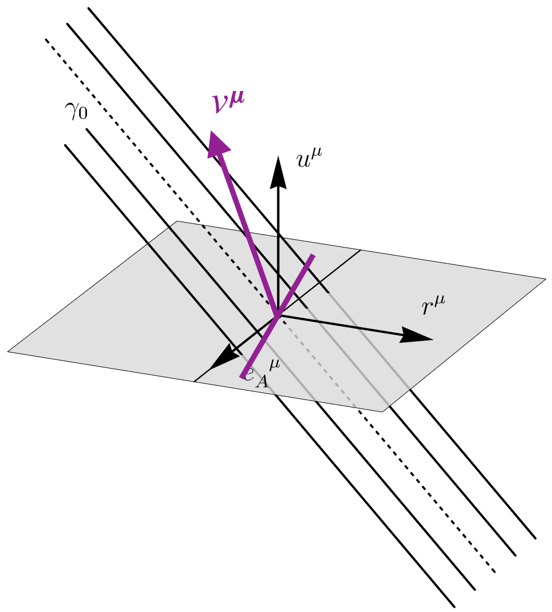
Yet, the distances measured to a given light rays (+ angles) are *the same*

Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

$$\delta x^\mu_{\mathcal{O}}\, l_\mu = 0$$



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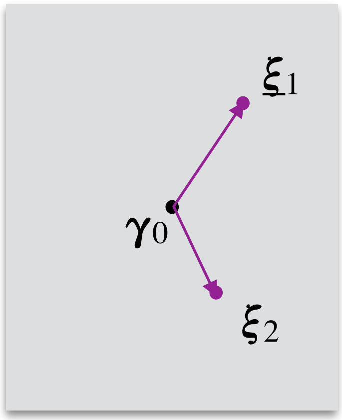
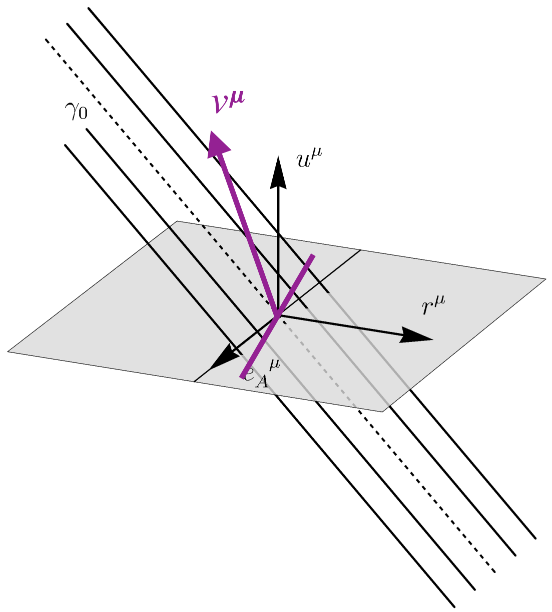


Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

$$\delta x^\mu_{\mathcal{O}}\, l_\mu = 0$$



Different observers u and v ⇒ different notions of simultaneity

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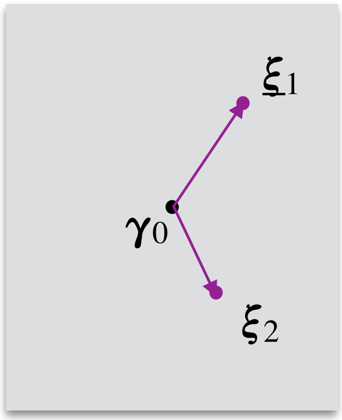
Yet, the distances measured to a given light rays (+ angles) are *the same*

$$g_{\mu\nu}\, \underline{\xi_1}^\mu\, \underline{\xi_2}^\nu = g_{AB}\, \xi_1{}^A\, \xi_2{}^B$$

$\mathscr{P}$ $\mathscr{P}$ $\mathscr{P}$



Geometry

Perpendicular space $\mathscr{P}$

- orthogonally displaced null geodesics

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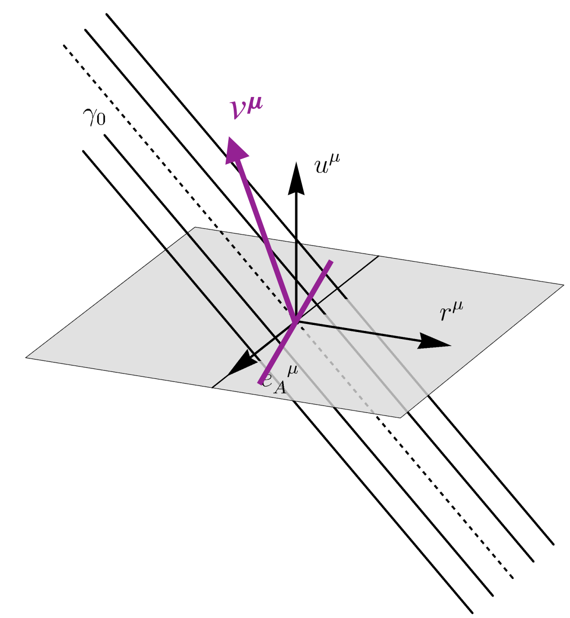

Different observers u and v ⇒ different notions of simultaneity

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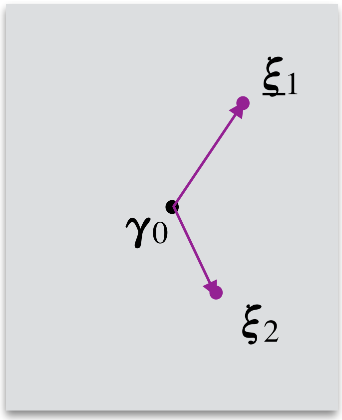
Yet, the distances measured to a given light rays (+ angles) are *the same*

$$g_{\mu\nu}\, \underline{\xi_1}^{\mu}\, \underline{\xi_2}^{\nu} = g_{AB}\, \xi_1{}^{A}\, \xi_2{}^{B}$$

$\mathscr{P}$ ↗ $\mathscr{P}$ ↑ $\mathscr{P}$ ↖

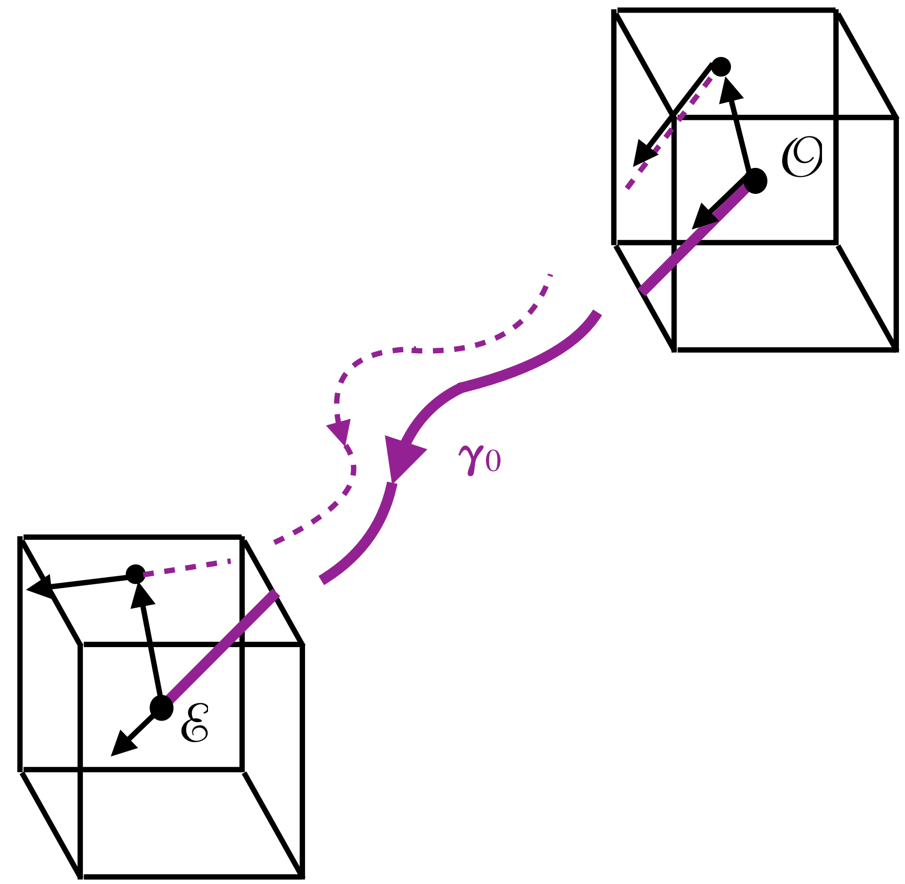


$\mathscr{P}$ = identification of screen spaces of all observers

Displacement formula

$$\delta x_{\mathcal{O}}^{\mu}\, l_{\mu} = \delta x_{\mathcal{E}}^{\mu}\, l_{\mu}$$

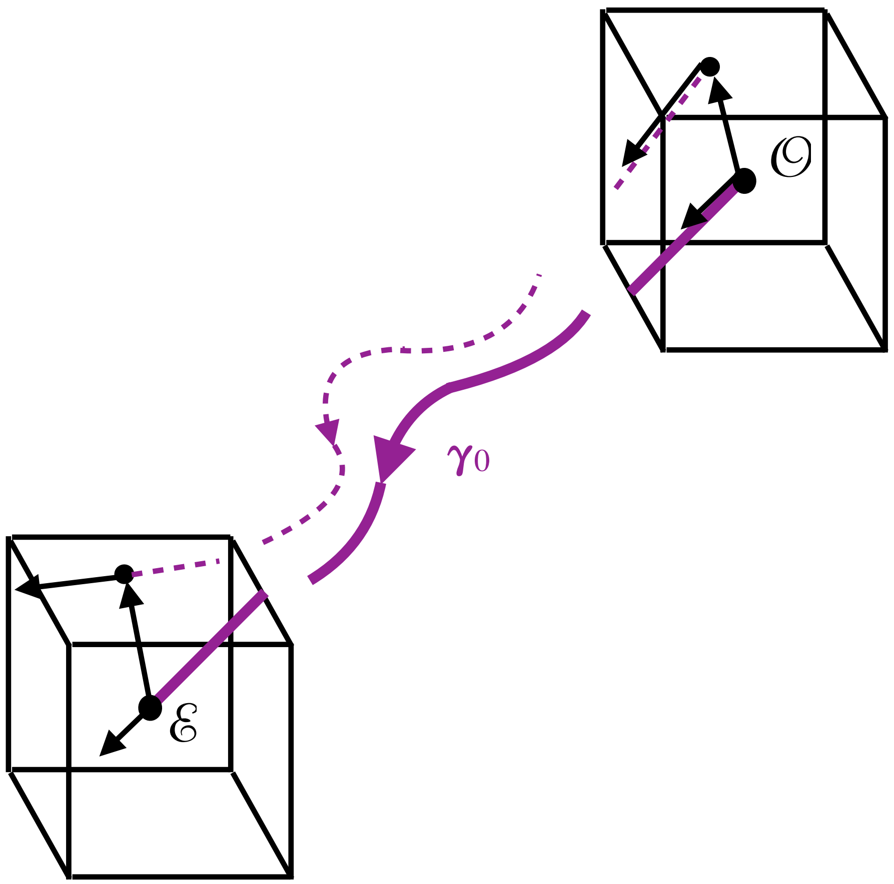
$$\delta x_{\mathcal{E}}^{\mu} \quad = \quad W_{xx}{}^{\mu}{}_{\nu}\, \delta x_{\mathcal{O}}^{\nu} + W_{xl}{}^{\mu}{}_{\nu}\, \Delta l_{\mathcal{O}}^{\nu}$$



Displacement formula

$$g([\delta x_{\mathcal{O}}], l) = g([\delta x_{\mathcal{E}}], l)$$

$$\mathcal{D}\left([\Delta l_{\mathcal{O}}]\right) = [\delta x_{\mathcal{E}} - \delta \hat{x}_{\mathcal{O}}] - m\left([\delta x_{\mathcal{O}}]\right)$$



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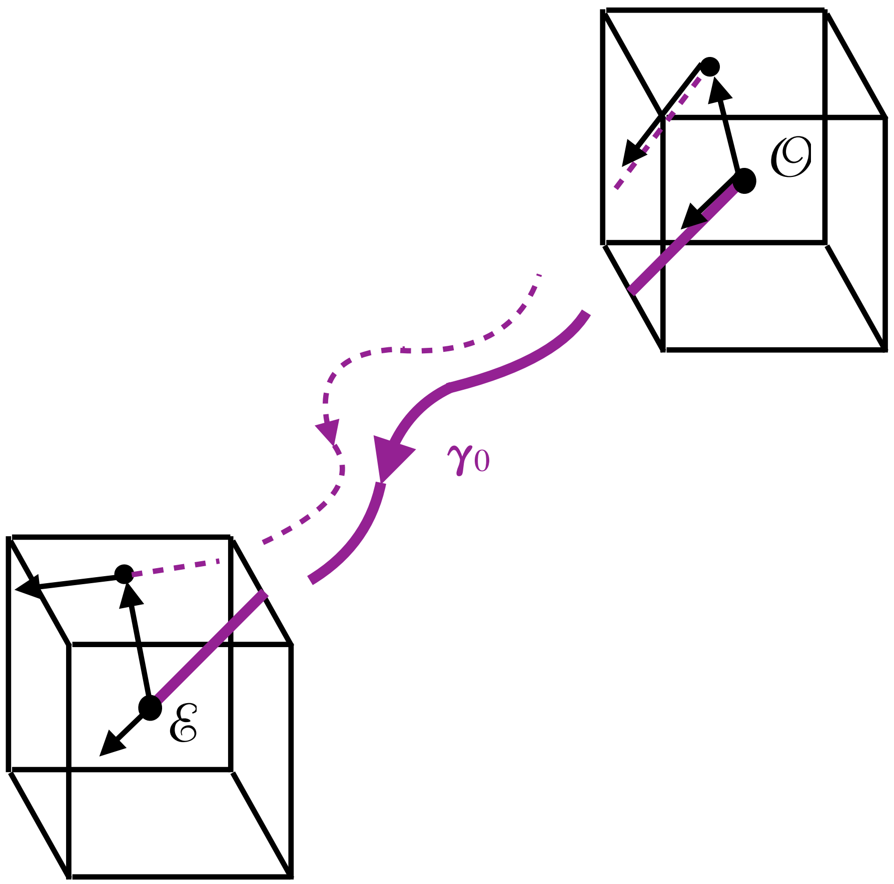
where

- ^ - parallel transport from $\mathcal{O}$ to $\mathcal{E}$



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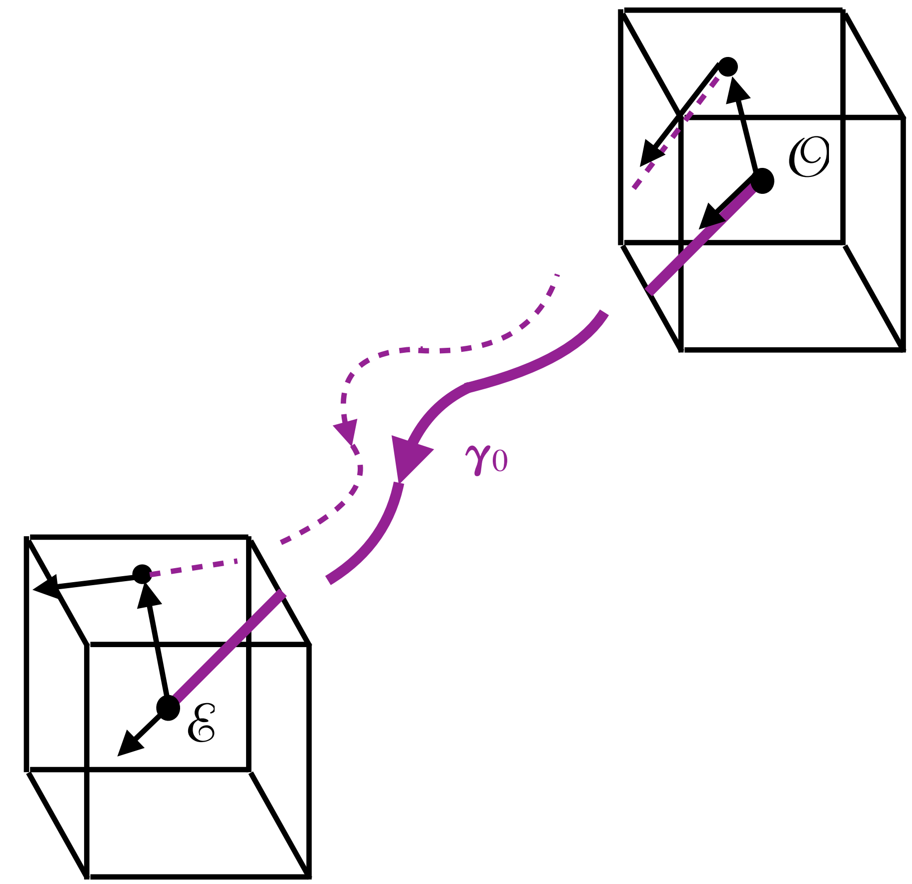
where

- ^ - parallel transport from $\mathcal{O}$ to $\mathcal{E}$
- Jacobi operator $\mathcal{D} : \mathcal{P}_{\mathcal{O}} \rightarrow \mathcal{P}_{\mathcal{E}}$

$$\ddot{\mathcal{D}}^{A}{}_{B} - R^{A}{}_{\mu\nu C}\, l^{\mu}\, l^{\nu}\, \mathcal{D}^{C}{}_{B} = 0$$
$$\mathcal{D}^{A}{}_{B}(\lambda_{\mathcal{O}}) = 0$$
$$\dot{\mathcal{D}}^{A}{}_{B}(\lambda_{\mathcal{O}}) = \delta^{A}{}_{B}$$



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$$\dot{\mathcal{D}}^{A}{}_{B}(\lambda_{\mathcal{O}}) = \delta^{A}{}_{B}$$

- $\mathcal{O}/\mathcal{E}$ asymmetry operator $m : \mathrm{T}_{\mathcal{O}}M\,/\,l \to \mathcal{P}_{\mathcal{E}}$

$$\ddot{m}^{A}{}_{\sigma} - R^{A}{}_{\mu\nu C}\, l^{\mu}\, l^{\nu}\, m^{C}{}_{\sigma} = R^{A}{}_{\mu\nu\sigma}\, l^{\mu}\, l^{\nu}$$
$$m^{A}{}_{\mu}(\lambda_{\mathcal{O}}) = 0$$
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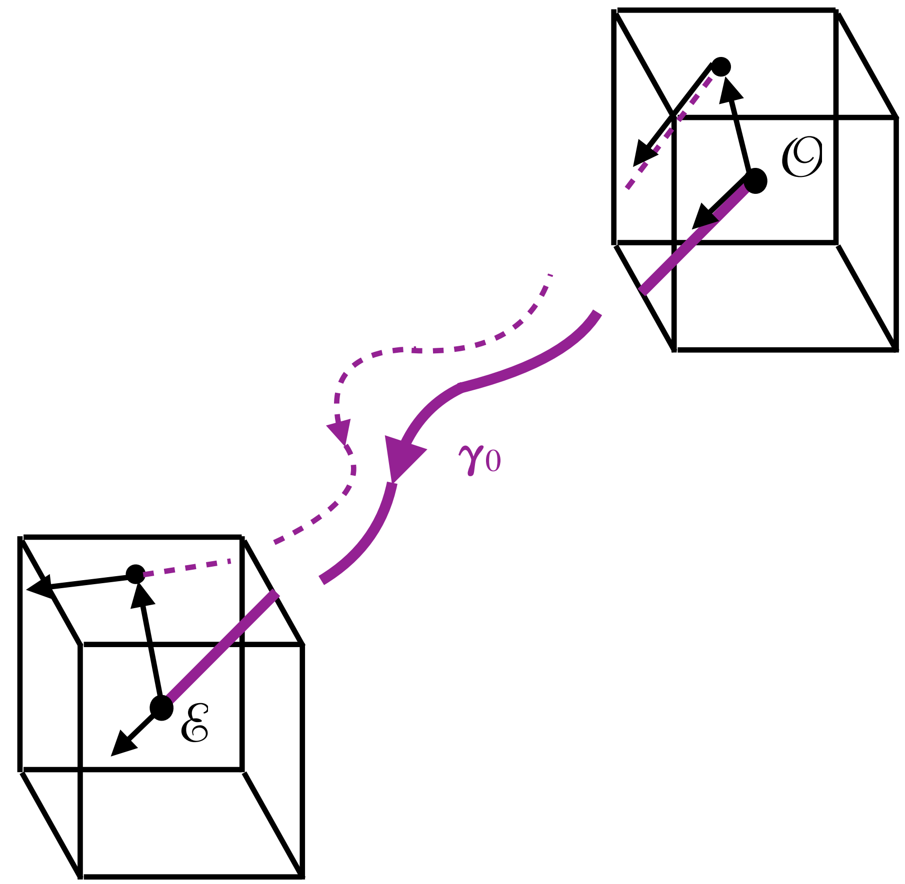

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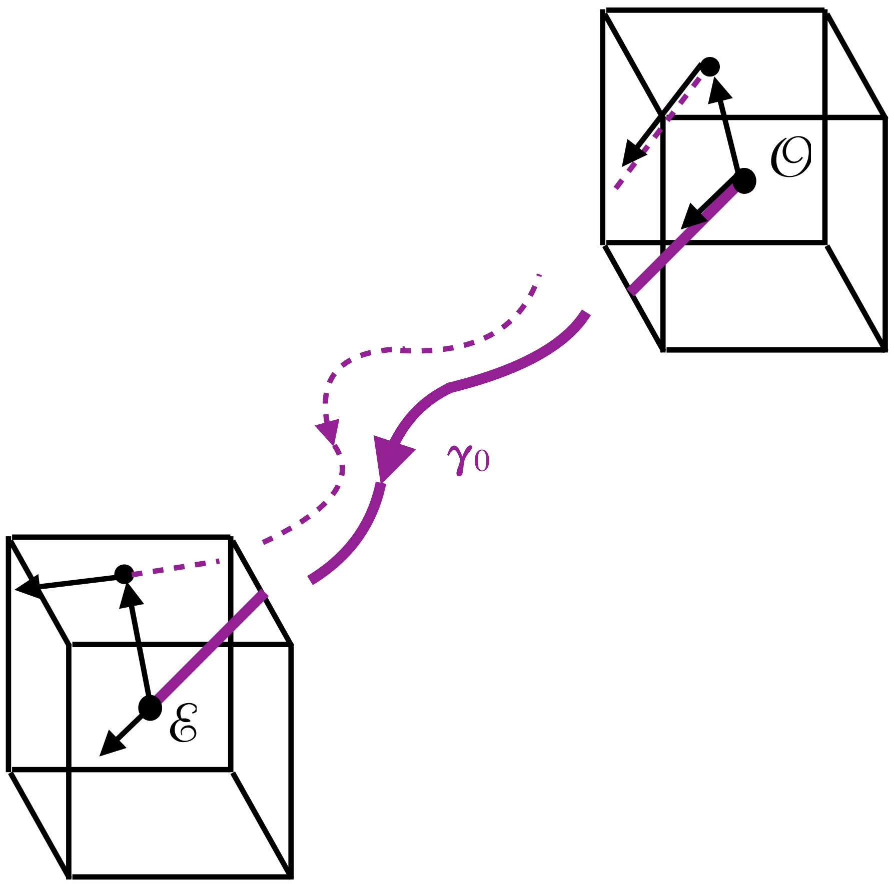

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$$m^A{}_\mu(\lambda_{\mathcal{O}}) = 0$$
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- vanishes in a flat space!

Displacement formula

$$g([\delta x_{\mathcal{O}}], l) = g([\delta x_{\mathcal{E}}], l)$$

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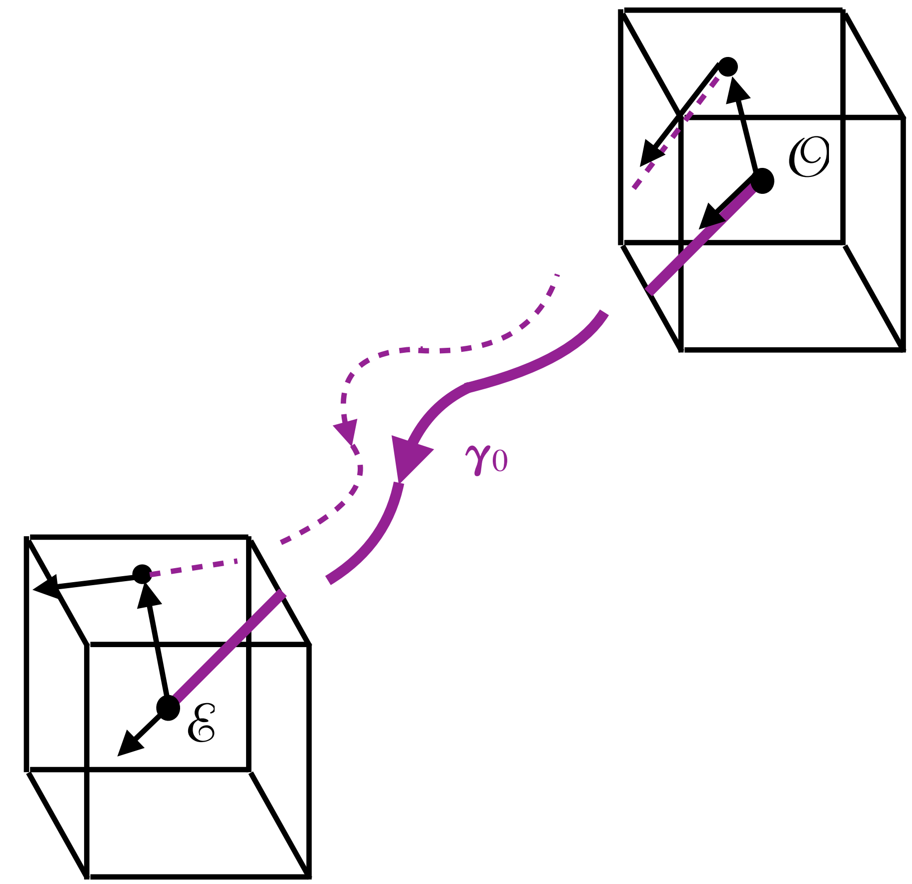

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$$\dot{m}^A{}_\mu(\lambda_{\mathcal{O}}) = 0$$

- vanishes in a flat space!

covariant, frame- and coordinate system-independent description of how observers in $N_{\mathcal{O}}$ see what is happening in $N_{\mathcal{E}}$

Displacement formula

$$g([\delta x_{\mathcal{O}}], l) = g([\delta x_{\mathcal{E}}], l)$$

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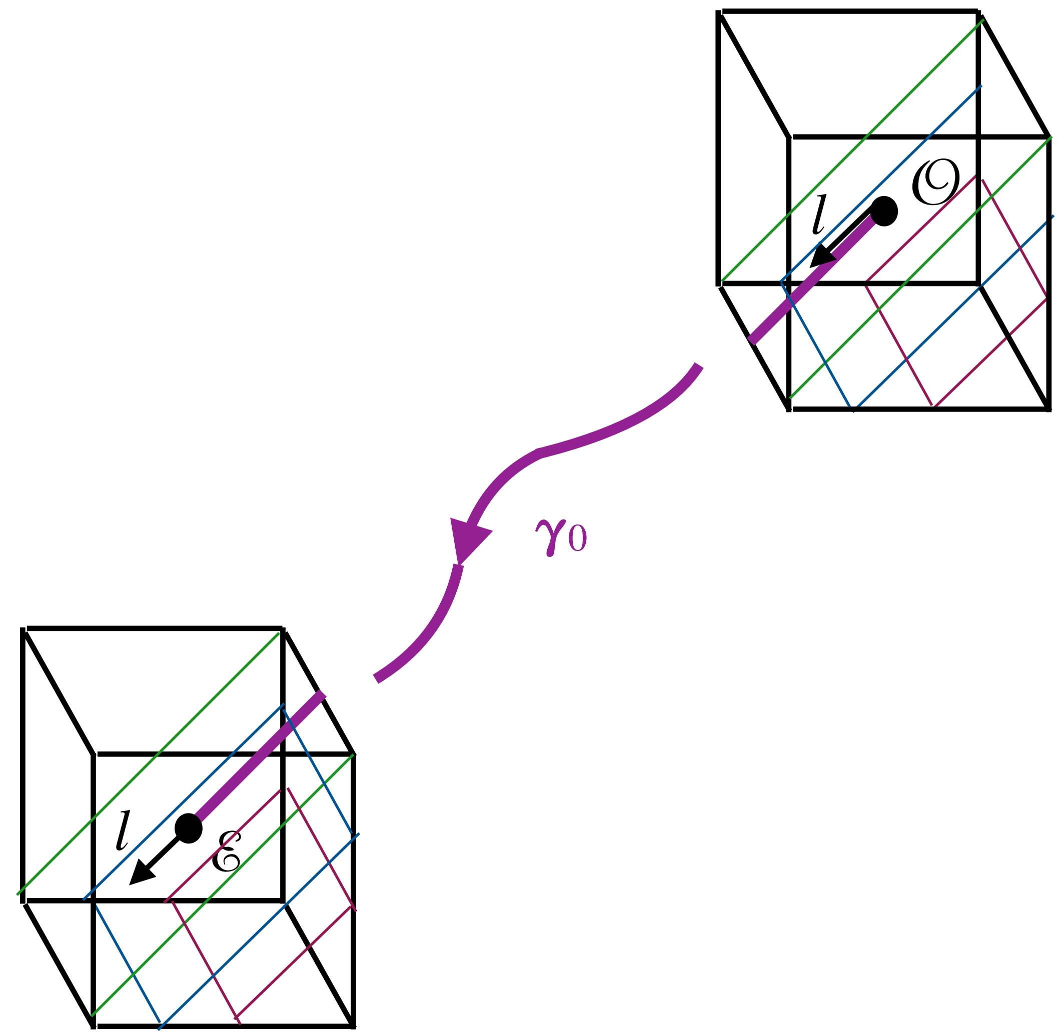

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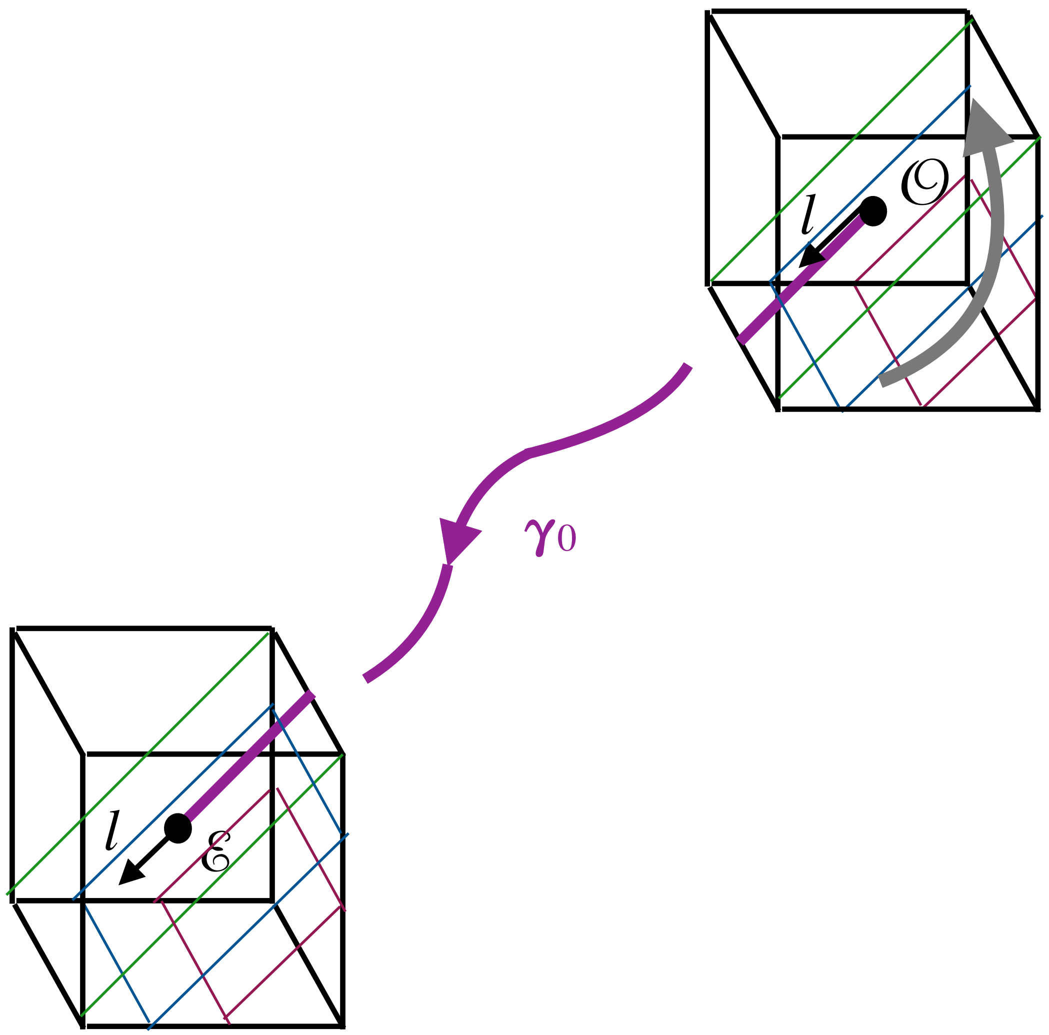

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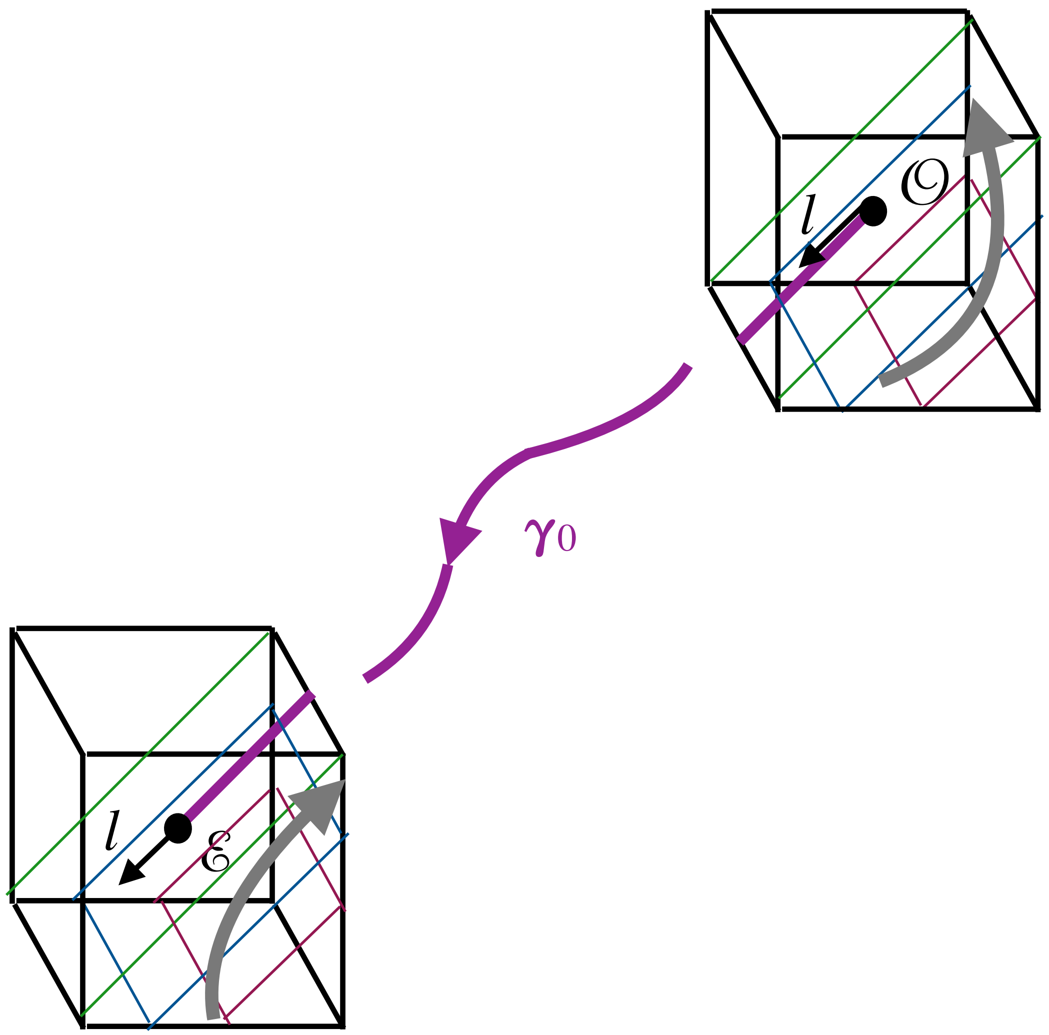

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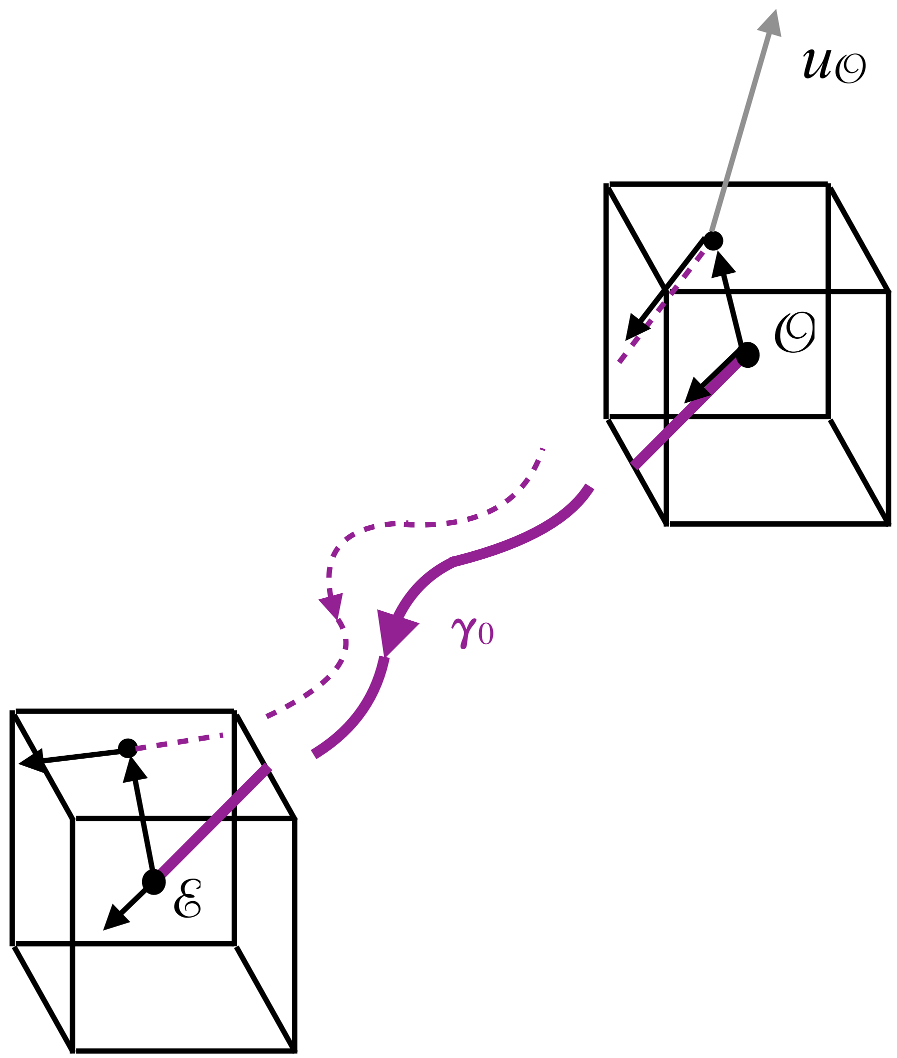

Displacement formula

$$g([\delta x_{\mathcal{O}}], l) = g([\delta x_{\mathcal{E}}], l)$$

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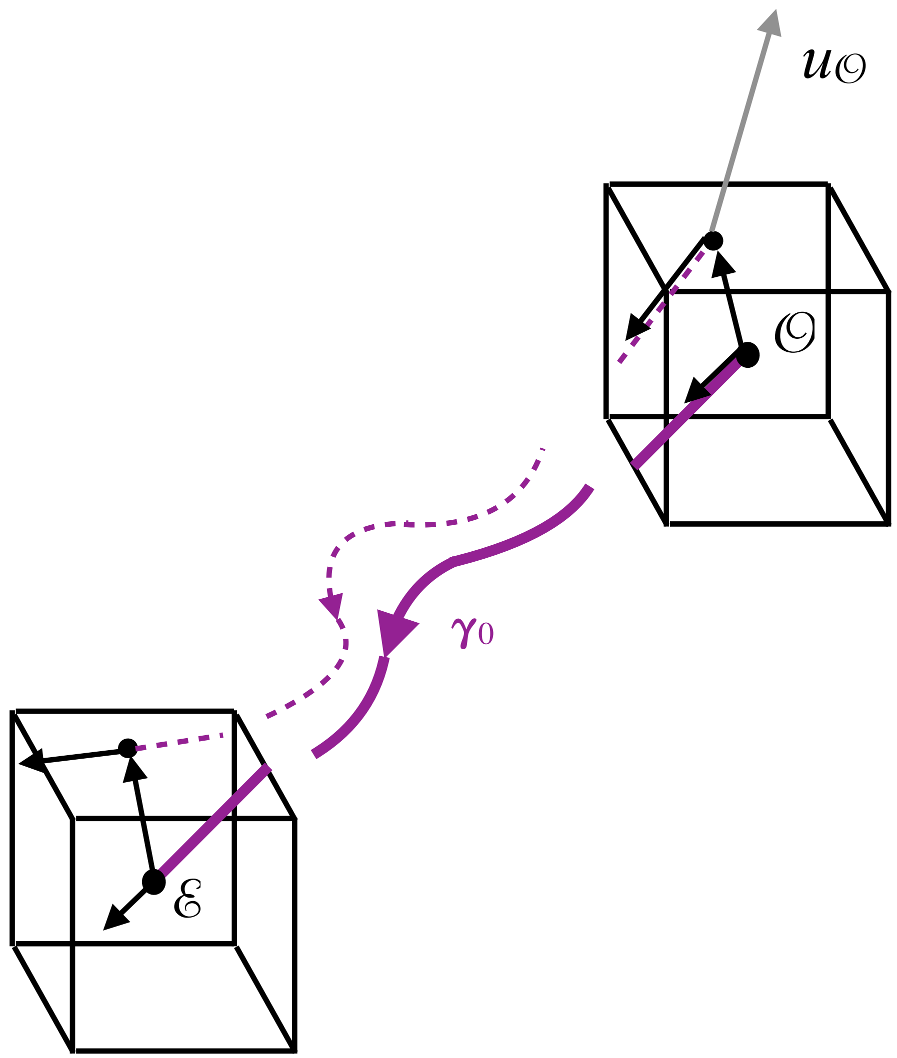
propagation through curved spacetime (GR)



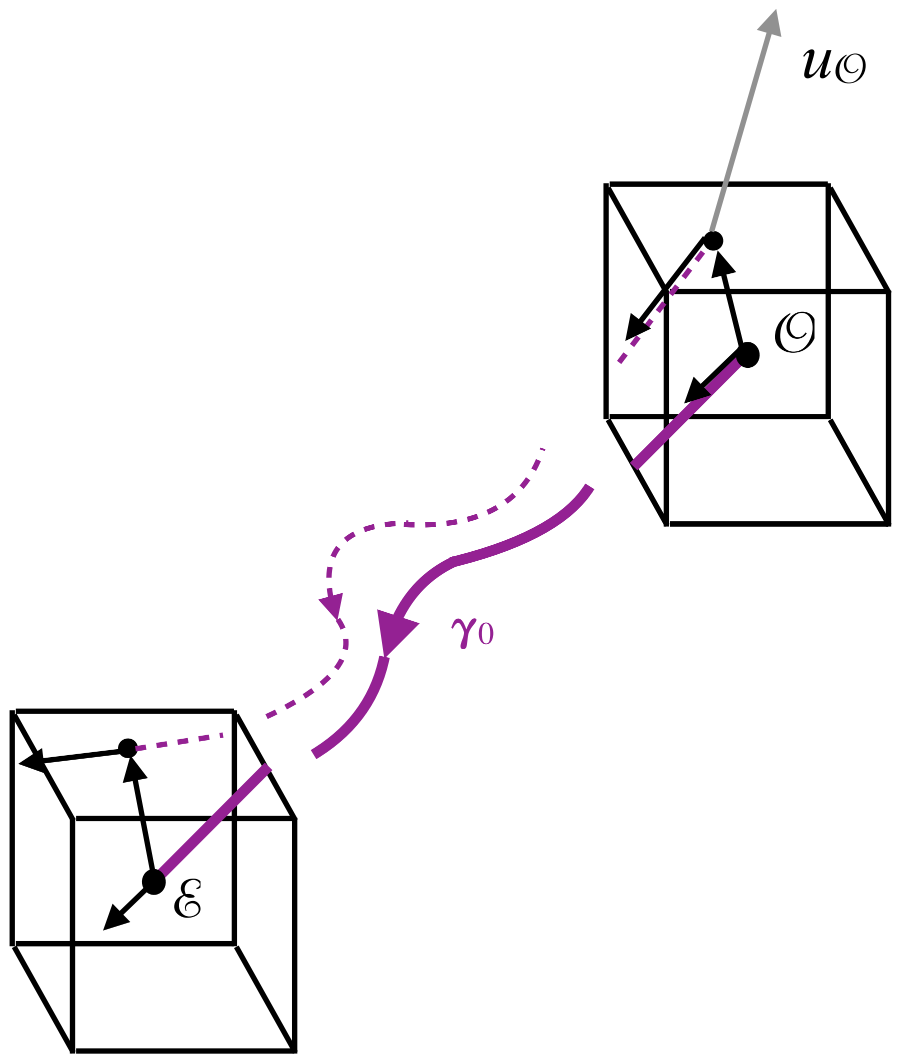
Displacement formula

$$g([\delta x_{\mathcal{O}}], l) = g([\delta x_{\mathcal{E}}], l)$$

$$\mathcal{D}\left([\Delta l_{\mathcal{O}}]\right) = [\delta x_{\mathcal{E}} - \delta \hat{x}_{\mathcal{O}}] - m\left([\delta x_{\mathcal{O}}]\right)$$

propagation through curved spacetime (GR)

angles on the celestial sphere of observer $u_{\mathcal{O}}$



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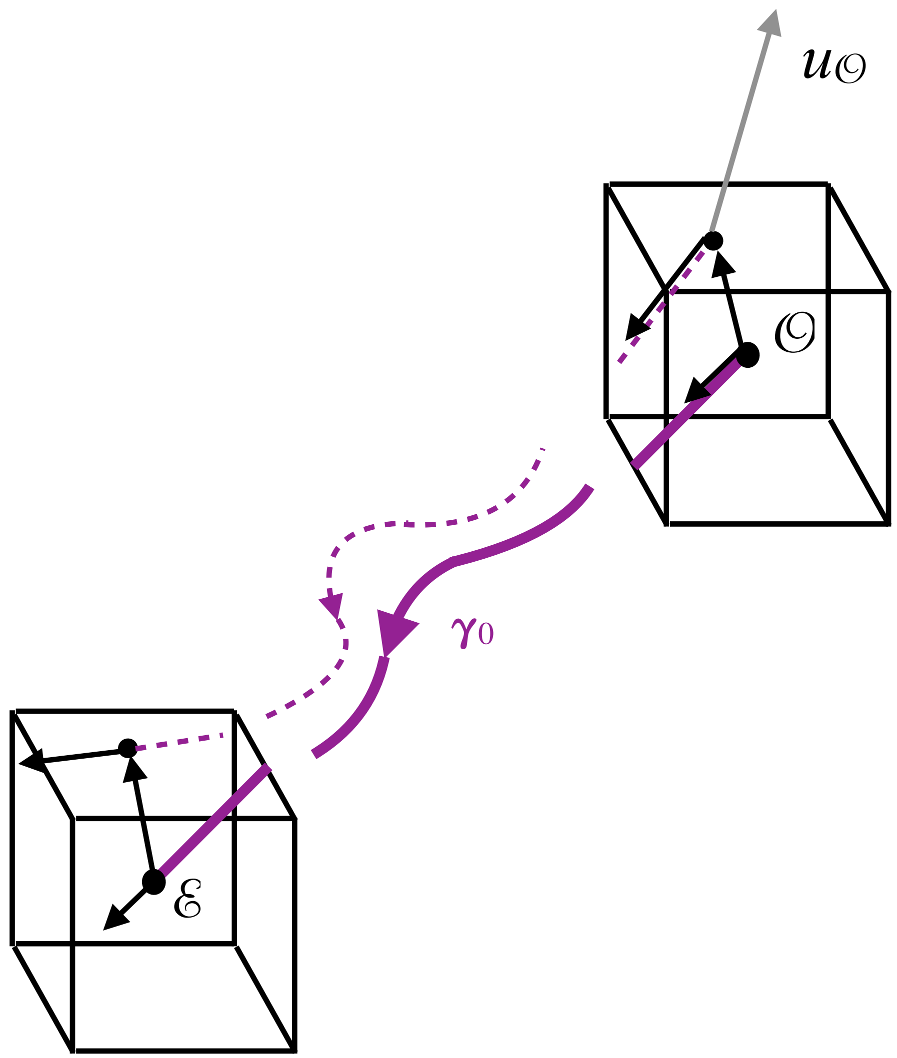

angles on the celestial sphere of observer $u_{\mathcal{O}}$

$$\delta\theta^A \approx \frac{1}{l_\sigma\, u^\sigma_{\mathcal{O}}}\,\Delta l^A$$

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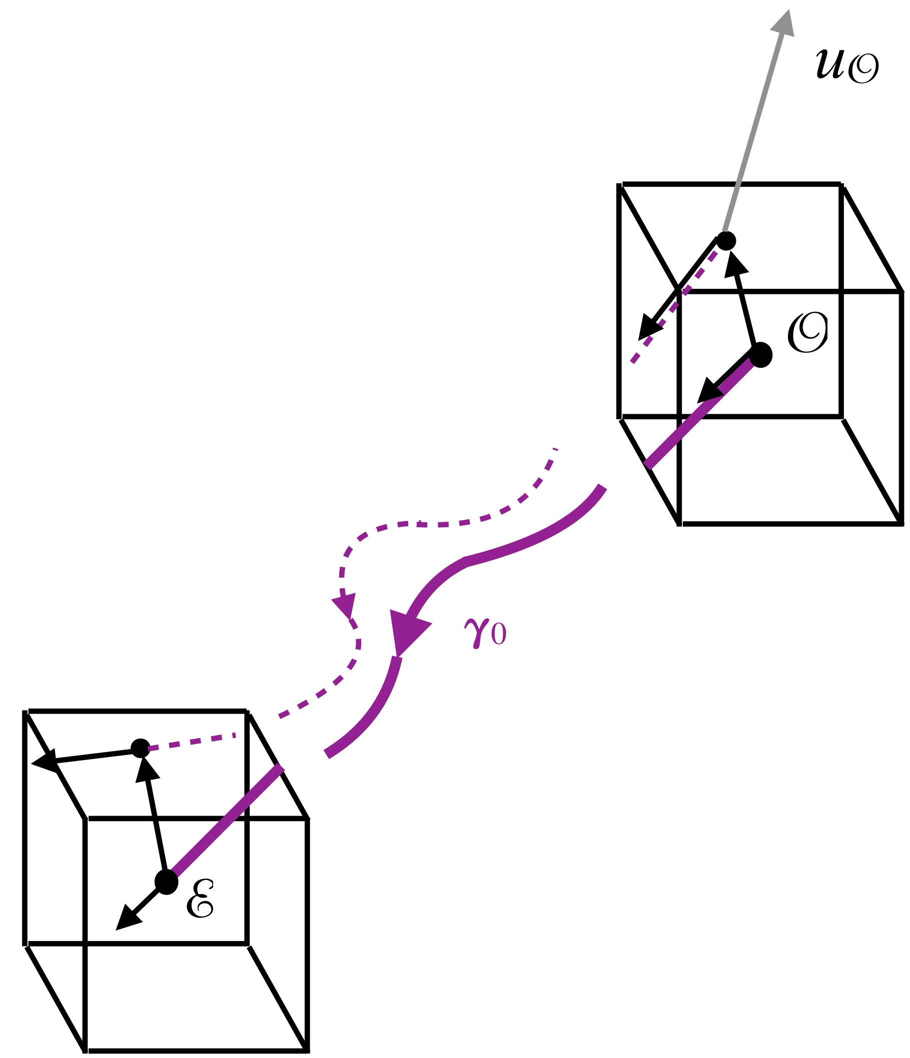
propagation through curved spacetime (GR)

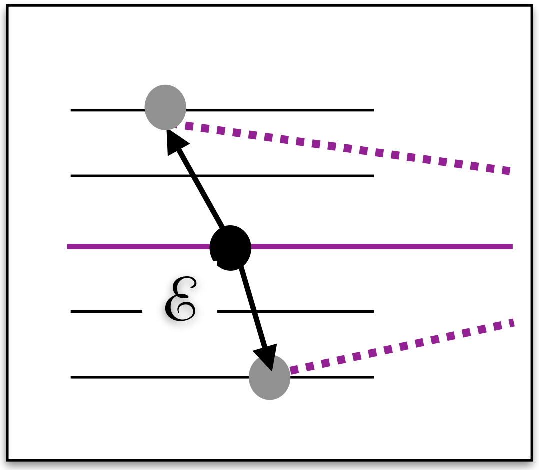


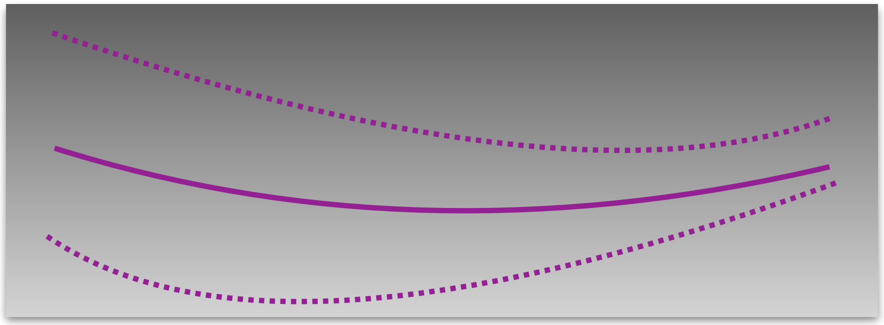
angles on the celestial sphere of observer $u_{\mathcal{O}}$

$$\delta\theta^A \approx \frac{1}{l_\sigma\, u^\sigma_{\mathcal{O}}}\, \Delta l^A$$

aberration effects (SR)

Image distortion





$$\mathcal{D}^A{}_B\,\Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$

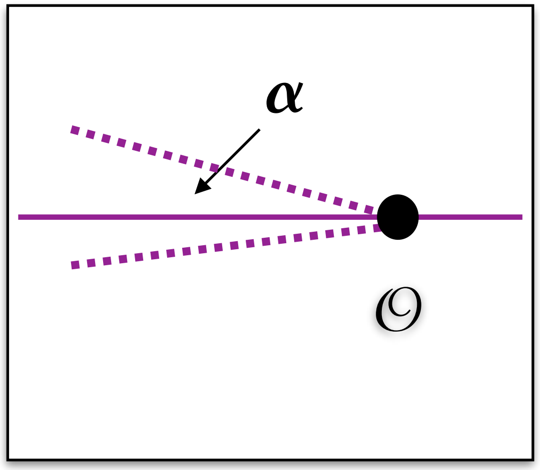
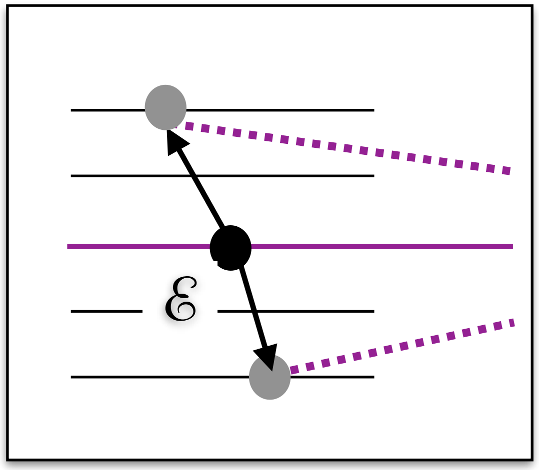
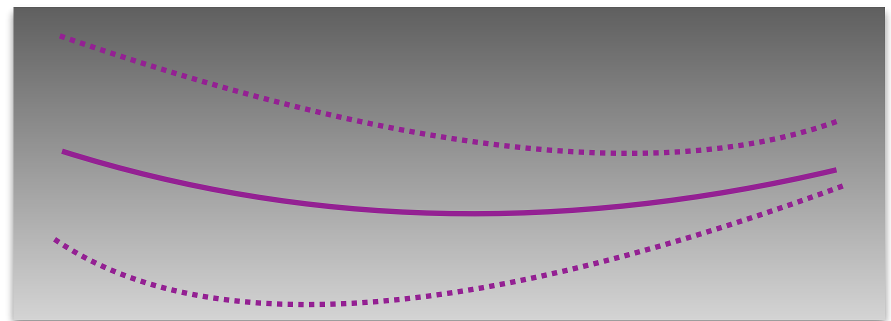
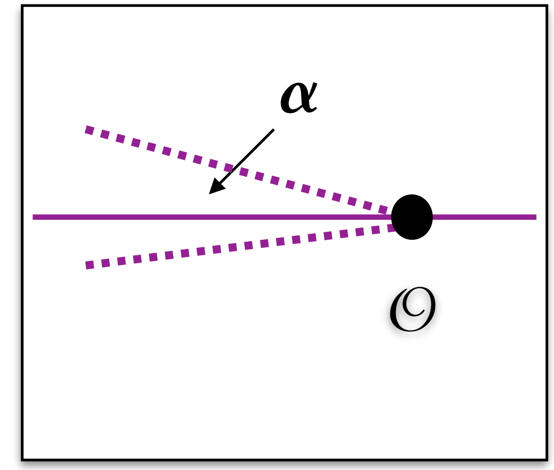


Image distortion



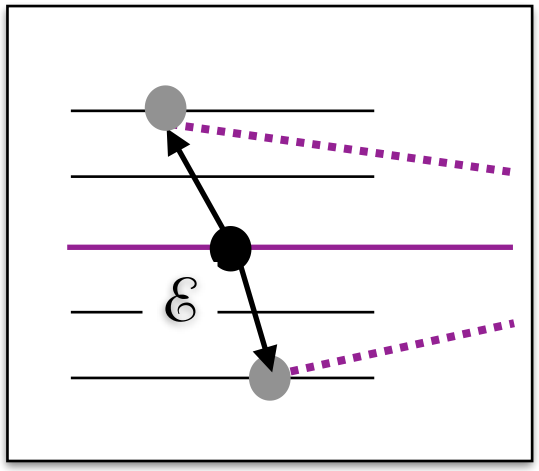


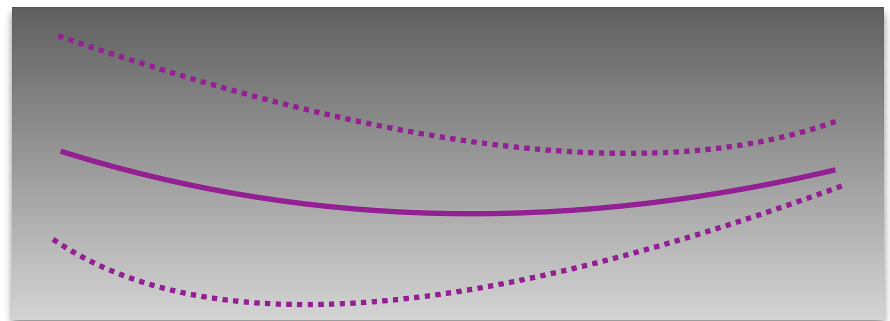
$$\mathcal{D}^A{}_B\,\Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$

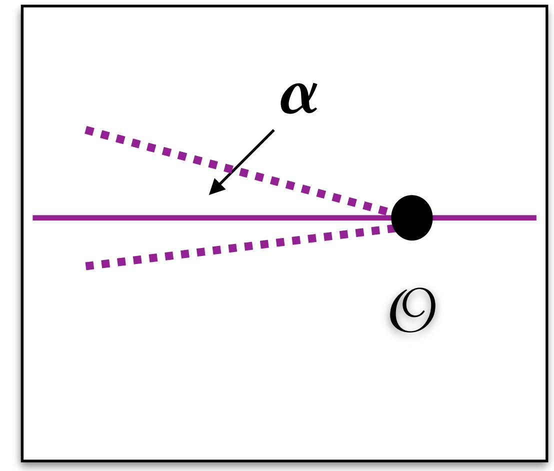


$$\delta\theta^A = \frac{1}{l_\sigma\, u^\sigma_{\mathcal{O}}}\,\mathcal{D}^{-1\,A}{}_B\,\delta x^B_{\mathcal{E}}$$

Image distortion







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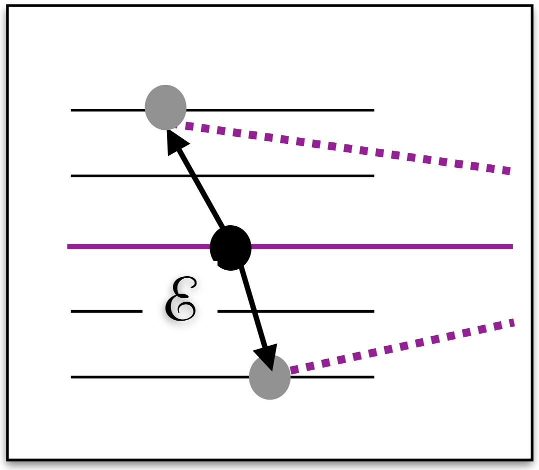
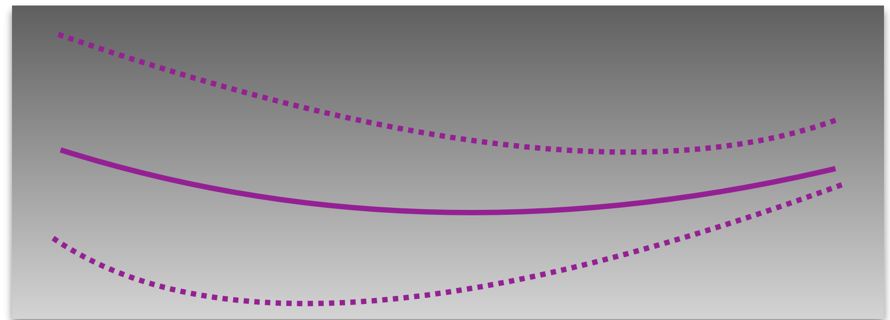
$M^A{}_B$ magnification matrix

Image distortion





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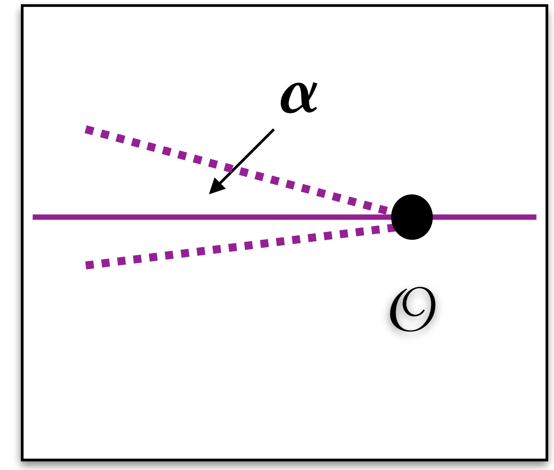

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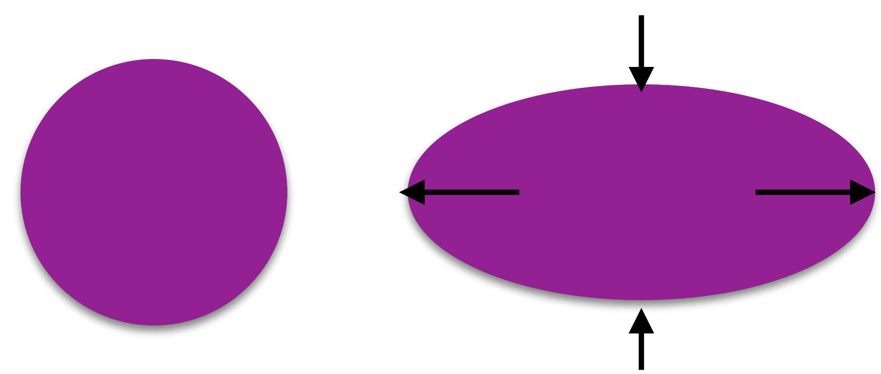
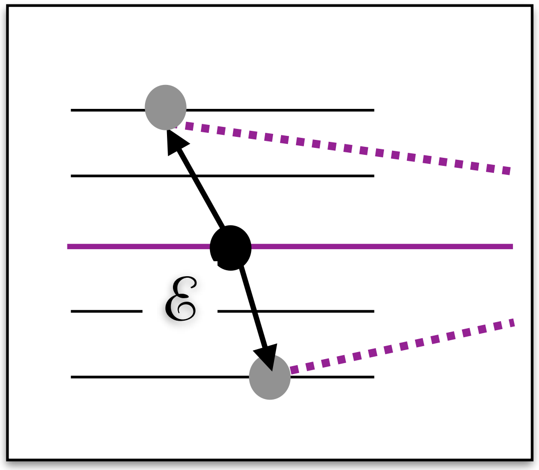
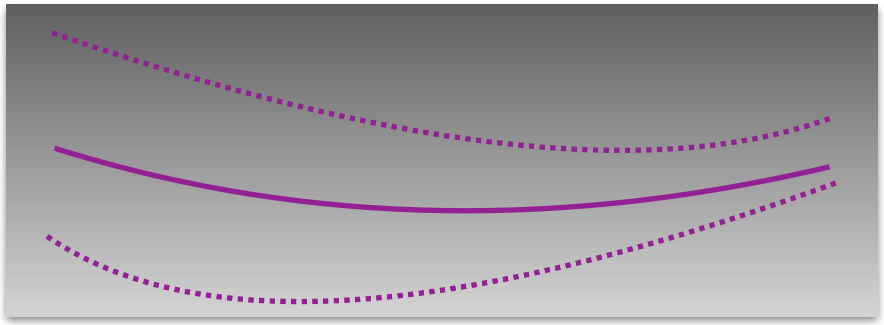


Image distortion





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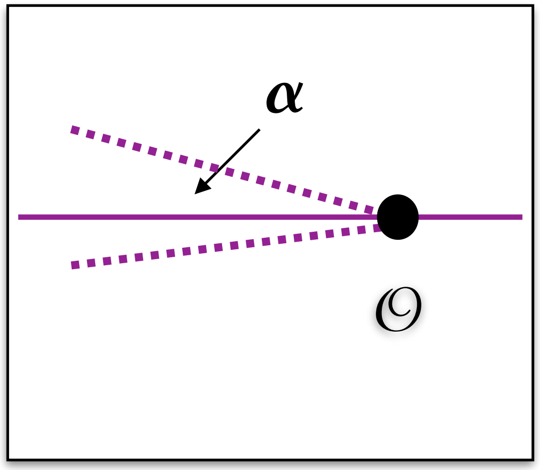

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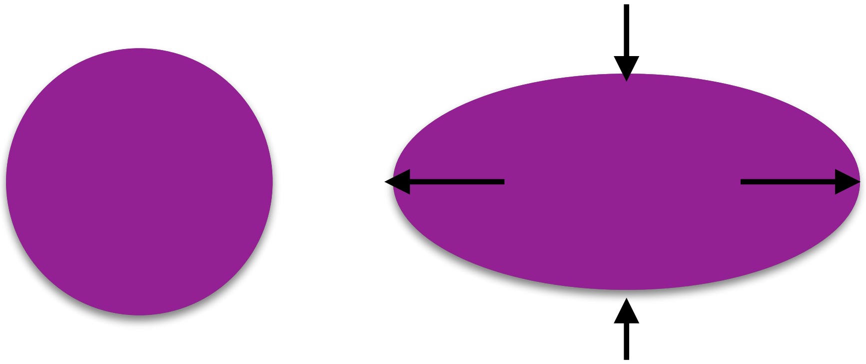
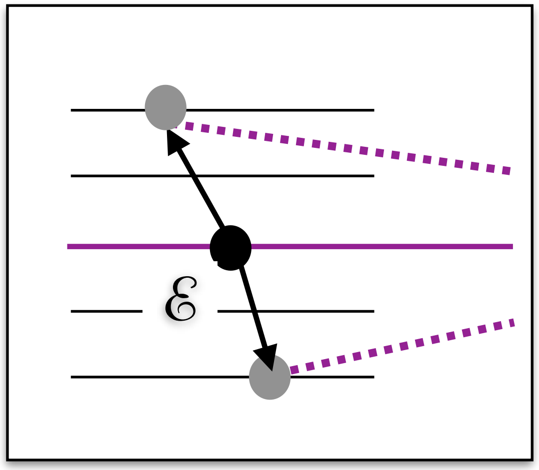
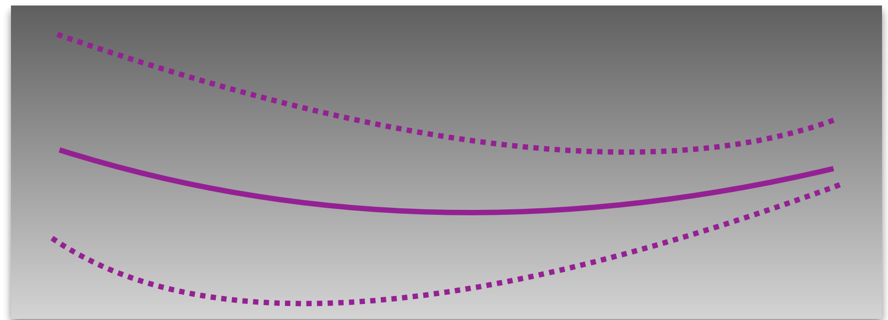
$M^A{}_B$ magnification matrix



$$D_{ang} = (l_\sigma\, u^\sigma_{\mathcal{O}})\left|\det \mathcal{D}^A{}_B\right|^{1/2}$$

Image distortion





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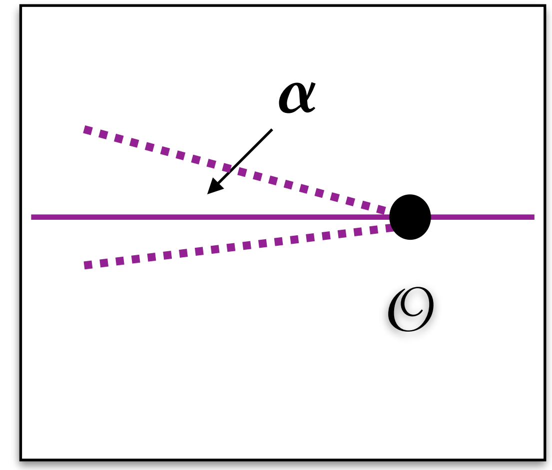

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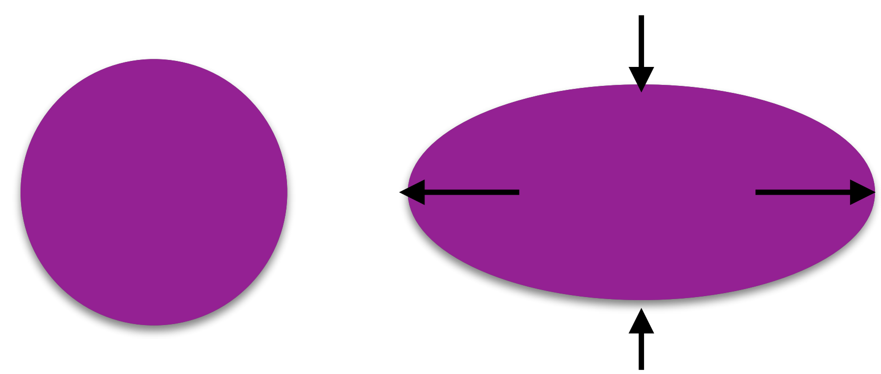
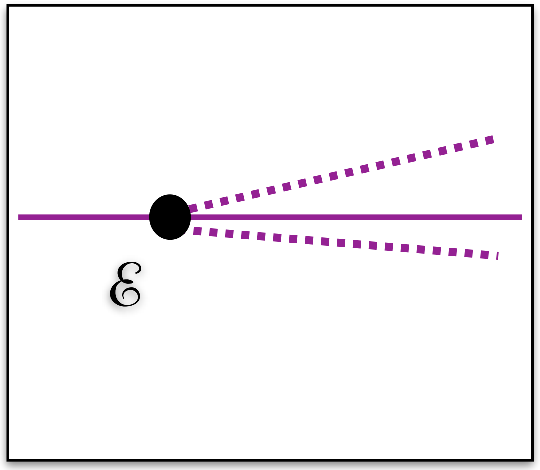
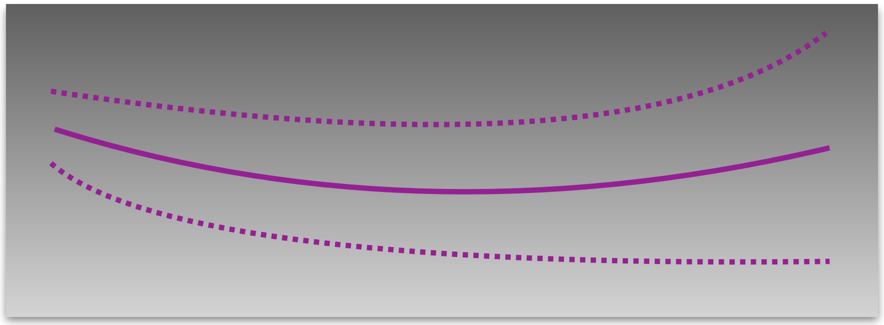


$$D_{ang} = (l_\sigma\, u^\sigma_{\mathcal{O}})\left|\det \mathcal{D}^A{}_B\right|^{1/2}$$

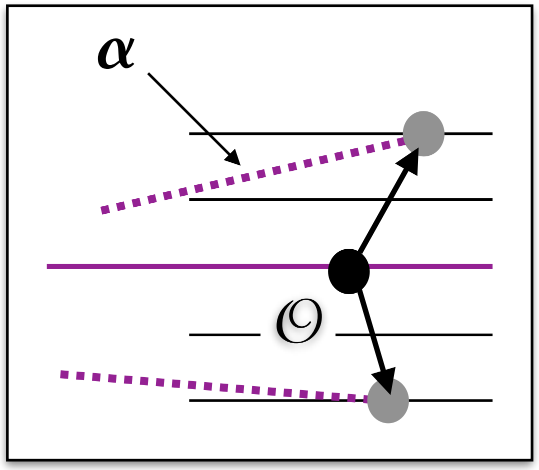
depend on $u_{\mathcal{O}}$, but NOT on $u_{\mathcal{E}}$!

Stereoscopic parallax





$$\mathcal{D}^A{}_B\,\Delta l^B_{\mathcal{O}} = -\delta\hat{x}^A_{\mathcal{O}} - m^A{}_B\,\delta x^B_{\mathcal{O}}$$



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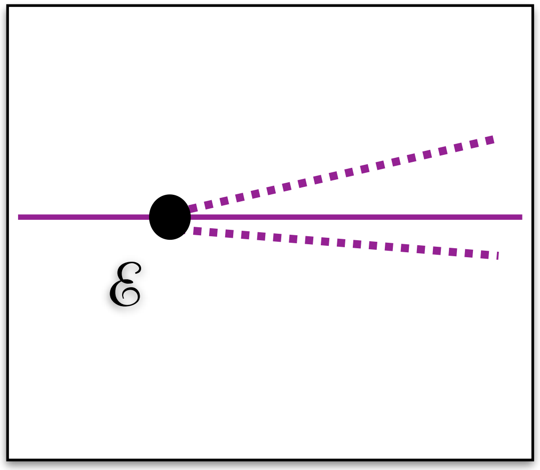
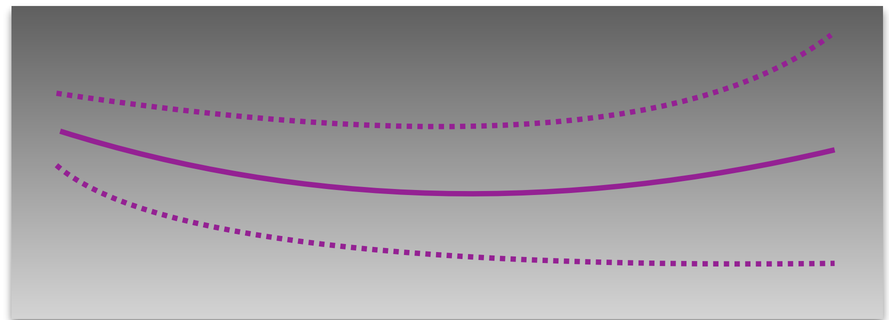
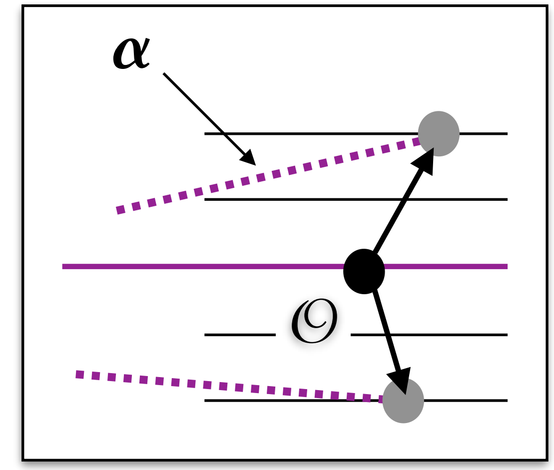


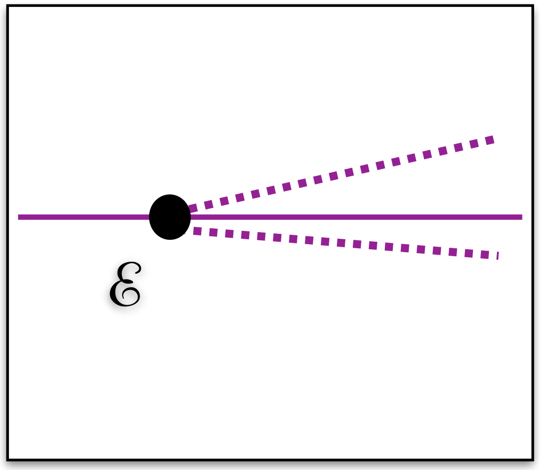


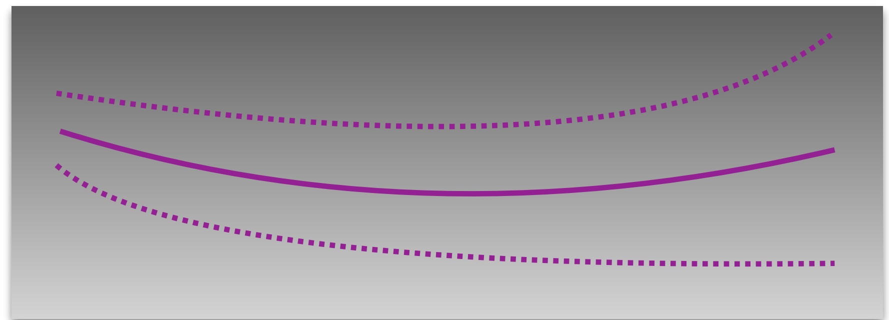


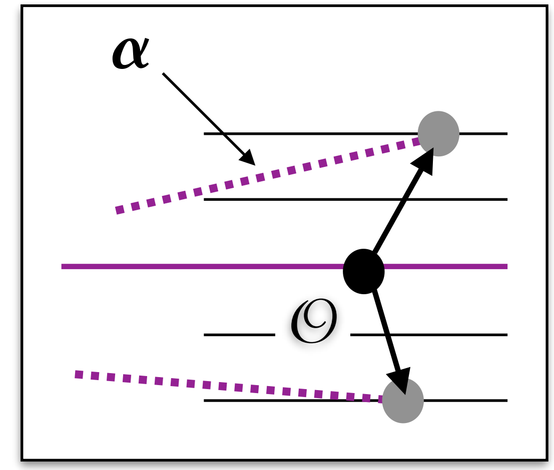
$$\mathcal{D}^A{}_B\,\Delta l^B_{\mathcal{O}} \;=\; -\delta\hat{x}^A_{\mathcal{O}} - m^A{}_B\,\delta x^B_{\mathcal{O}}$$

$$\delta\theta^A = -\frac{1}{l_\sigma\,u^\sigma_{\mathcal{O}}}\,\mathcal{D}^{-1\,A}{}_C\,(\delta^C{}_B + m^C{}_B)\,\delta x^B_{\mathcal{O}}$$

Stereoscopic parallax







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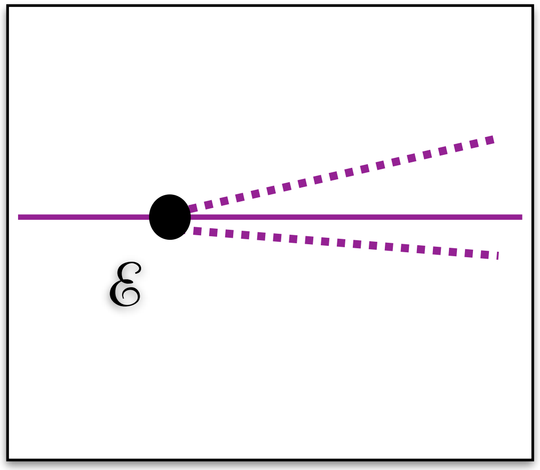
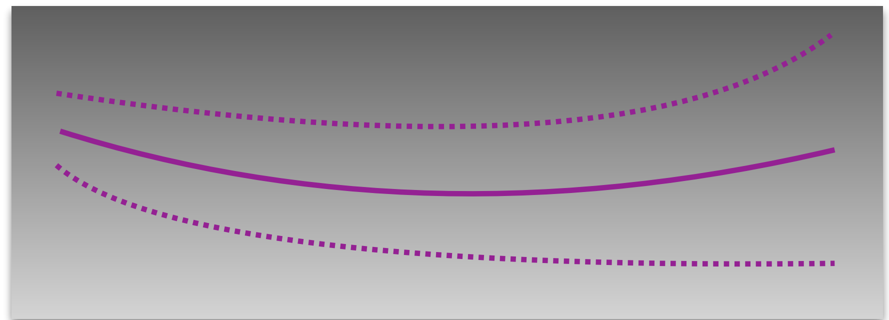
$\Pi^A{}_B$ parallax matrix

Stereoscopic parallax





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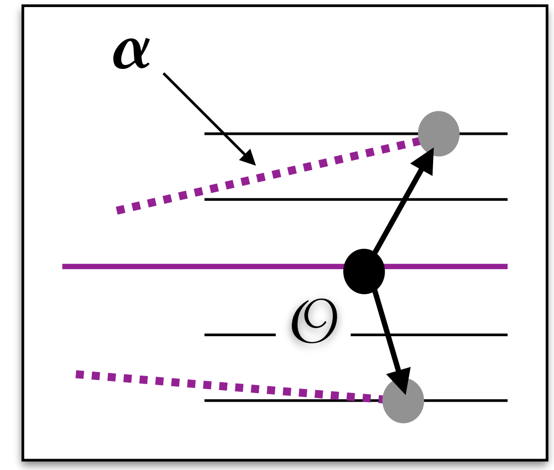
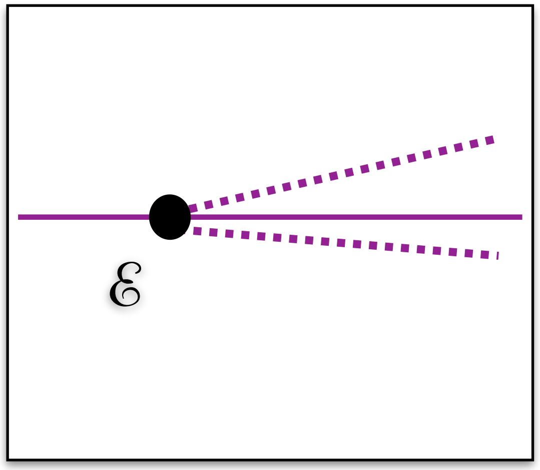


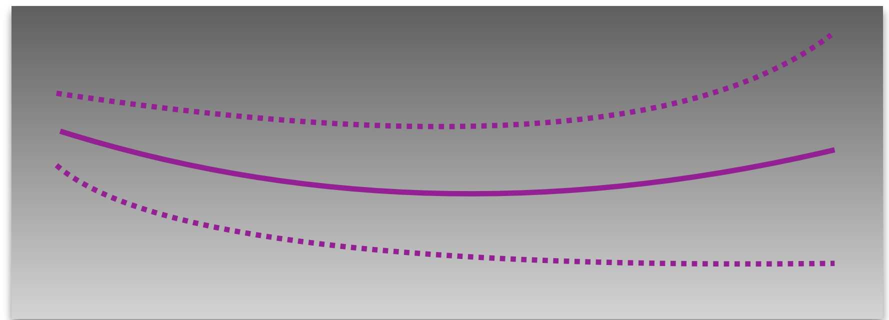
$$\delta\theta^A = -\frac{1}{l_\sigma\, u^\sigma_{\mathcal{O}}}\,\mathcal{D}^{-1\,A}{}_C\,(\delta^C{}_B + m^C{}_B)\,\delta x^B_{\mathcal{O}}$$

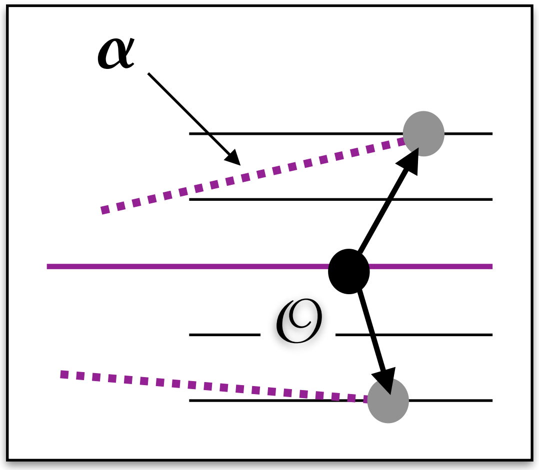
$\Pi^A{}_B$ parallax matrix

$$D_{par} = D_{ang}\left|\det\left(\delta^A{}_B + m^A{}_B\right)\right|^{-1/2}$$

Stereoscopic parallax







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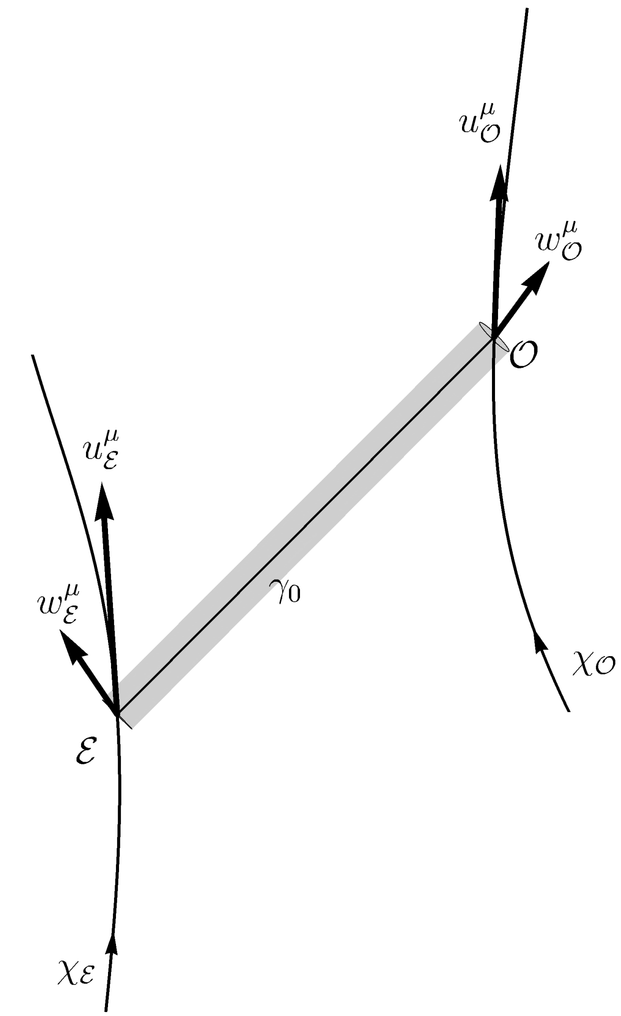
depend on $u_{\mathcal{O}}$, but NOT on $u_{\mathcal{E}}$!

Position drift



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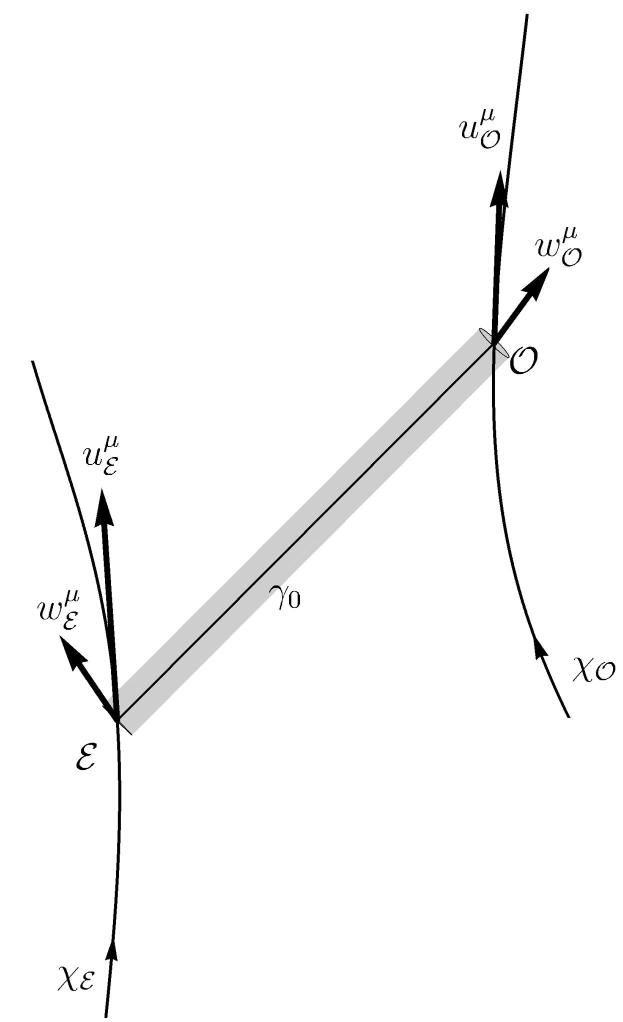

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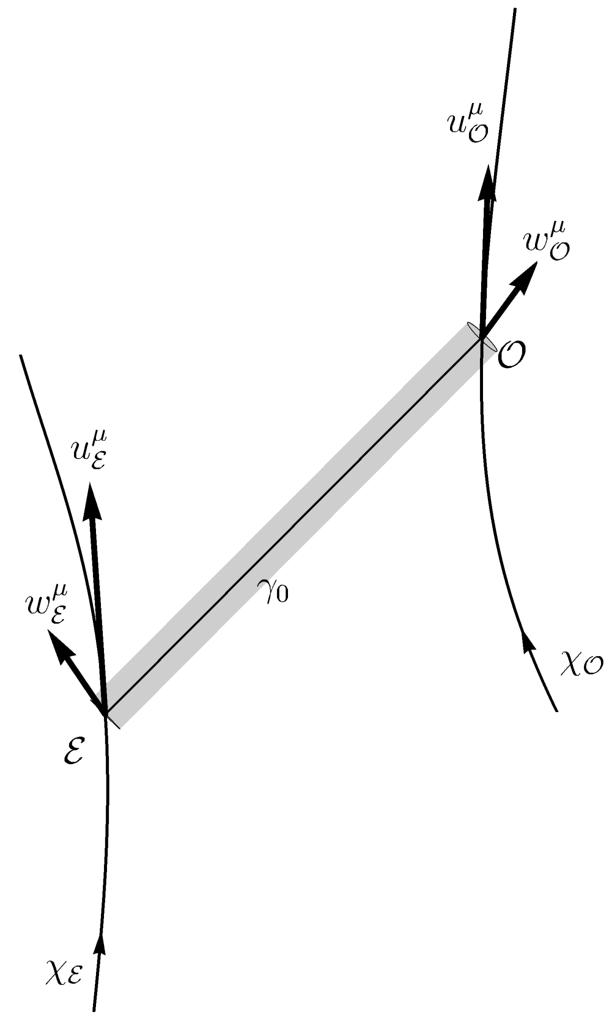
$$\frac{\mathrm{d}\tau_{\mathcal{O}}}{\mathrm{d}\tau_{\mathcal{E}}} = \frac{l_\sigma\, u^\sigma_{\mathcal{E}}}{l_\rho\, u^\rho_{\mathcal{O}}} = \frac{1}{1+z}$$



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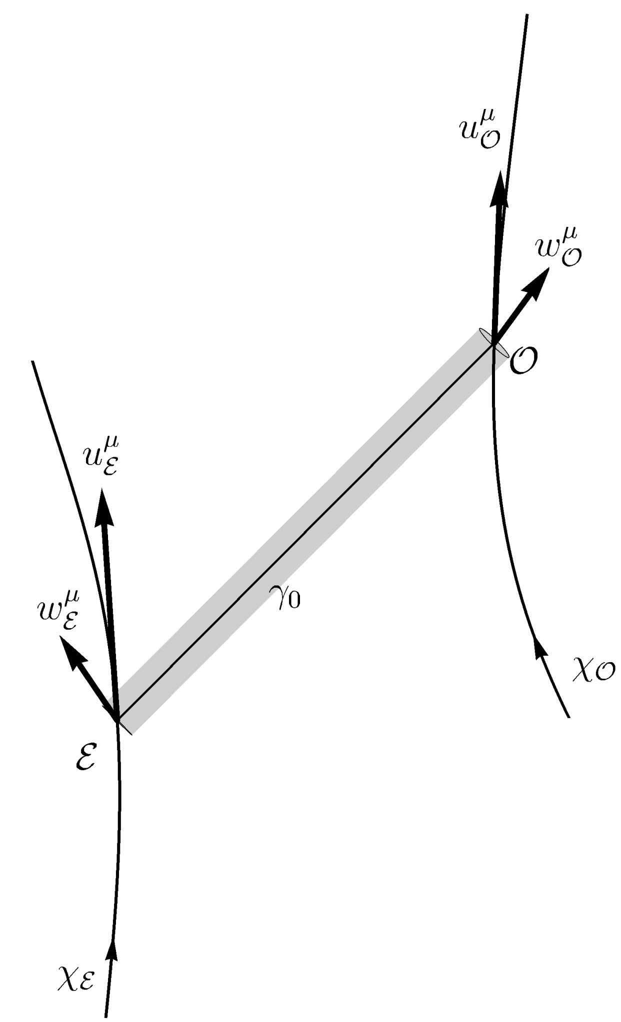
$$\frac{\Delta l_{\mathcal{O}}^A}{\mathrm{d}\tau_{\mathcal{O}}} = {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}}\right)^B - m^B{}_\mu\, u_{\mathcal{O}}^\mu\right)$$



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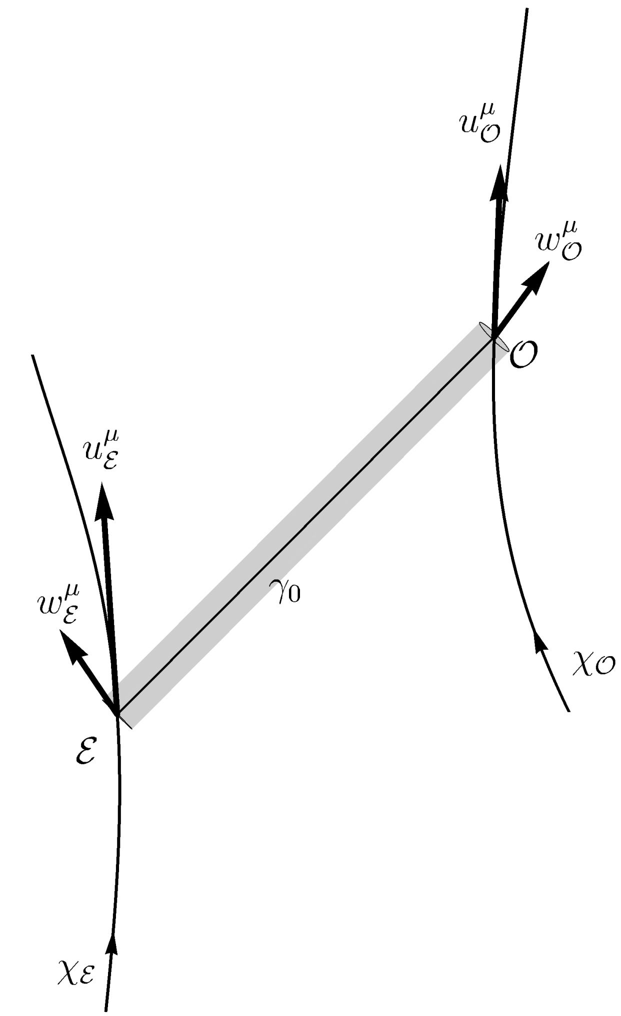

- Non-geodesic observer: calculate the Fermi-Walker derivative of the position vector on the celestial sphere r^μ

Position drift

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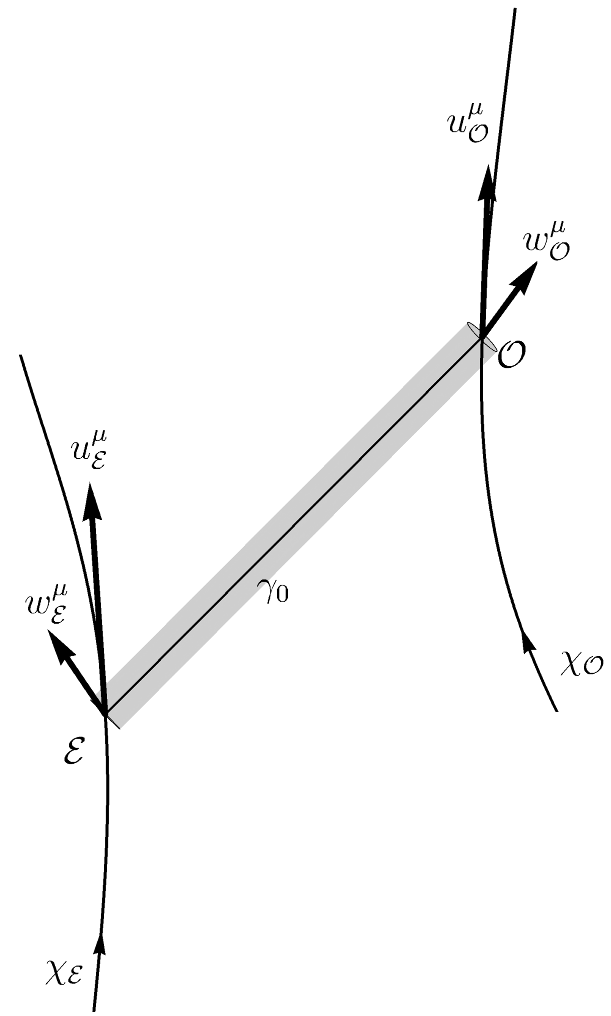

- Non-geodesic observer: calculate the Fermi-Walker derivative of the position vector on the celestial sphere r^μ

$$\delta_{\mathcal{O}} r^A = w_{\mathcal{O}}^A + \frac{1}{l_\sigma\, u_{\mathcal{O}}^\sigma}\, \mathcal{D}^{-1\,A}{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}}\right)^B - m^B{}_\mu\, u_{\mathcal{O}}^\mu\right)$$

Position drift

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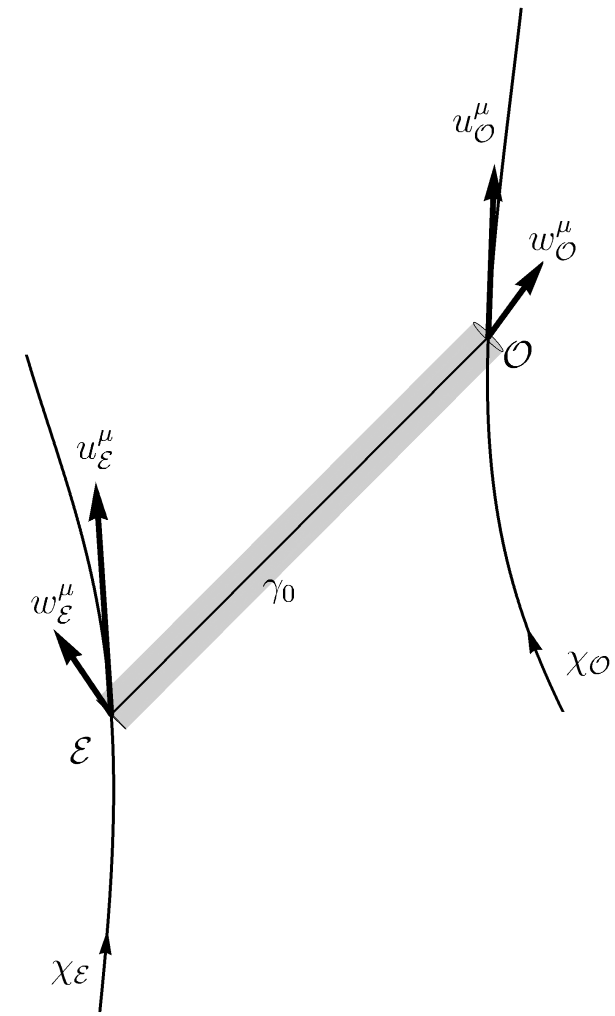
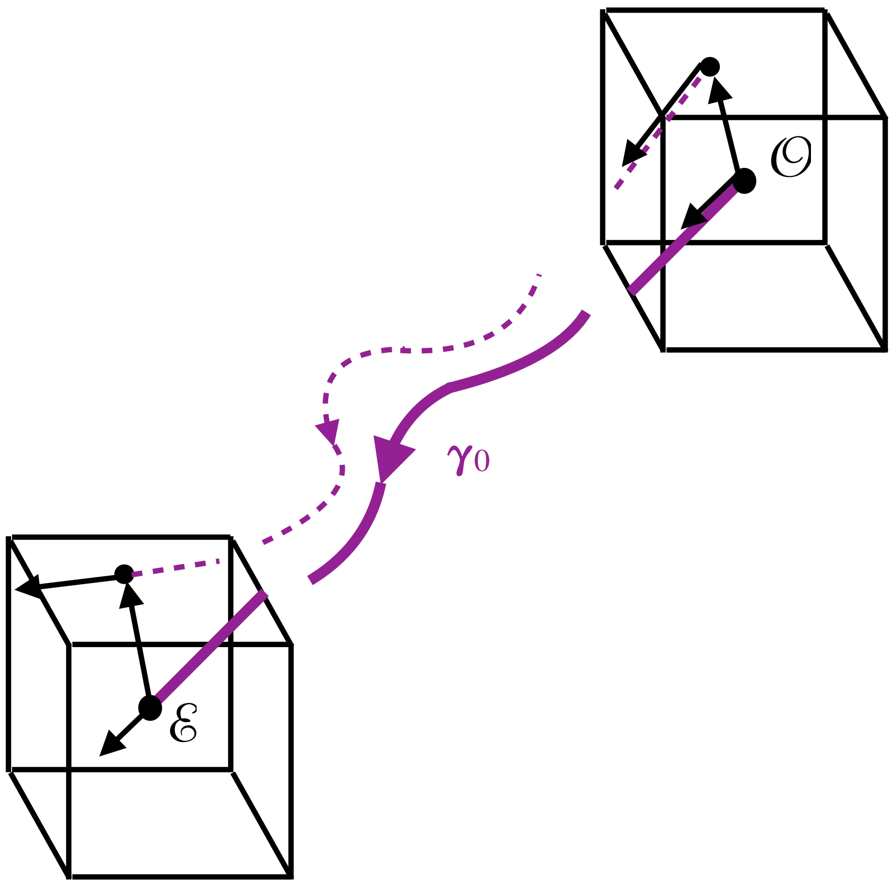


- Non-geodesic observer: calculate the Fermi-Walker derivative of the position vector on the celestial sphere r^μ

aberration drift

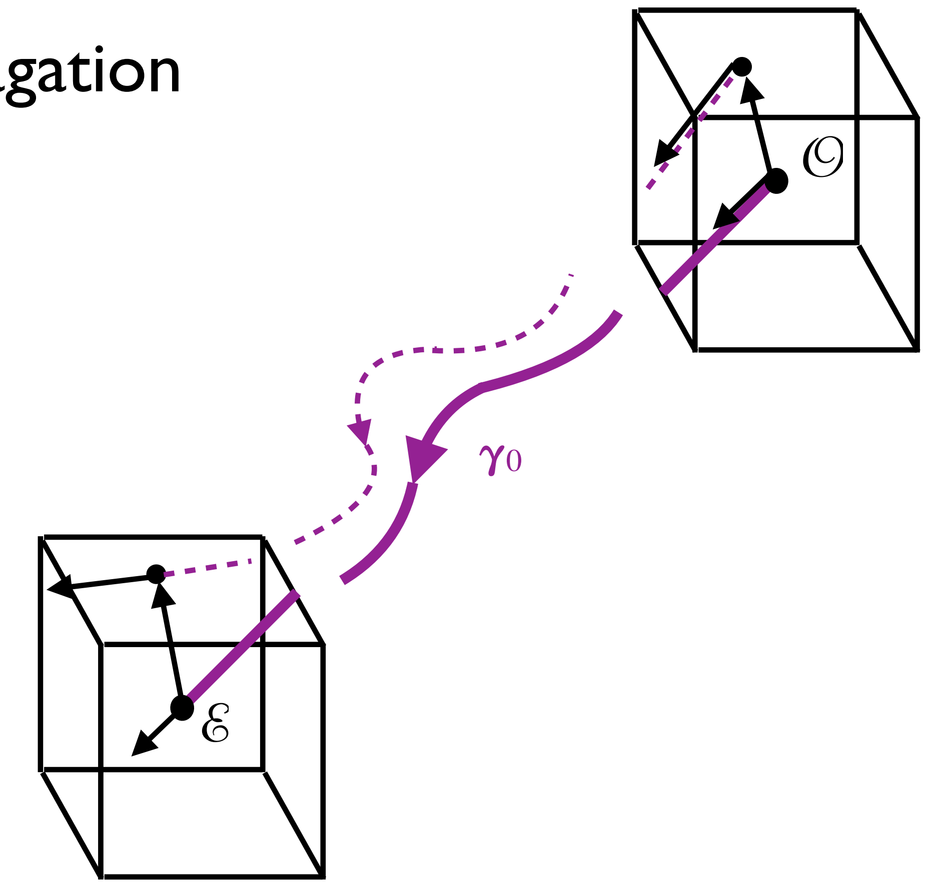
$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{l_\sigma\, u^\sigma_{\mathcal{O}}}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u^\mu_{\mathcal{O}} \right)$$

Summary



Summary

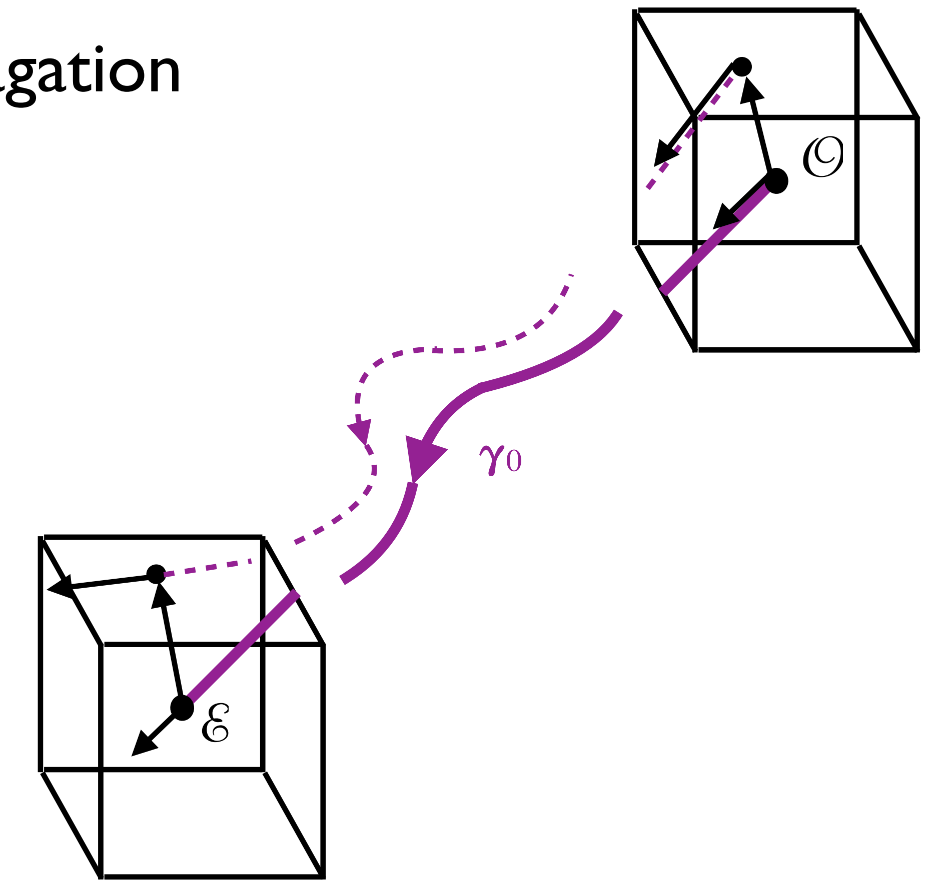
Covariant description of geometric optics and light propagation between small, distant regions



Summary

Covariant description of geometric optics and light propagation between small, distant regions

Valid in any spacetime (distant observer approximation)

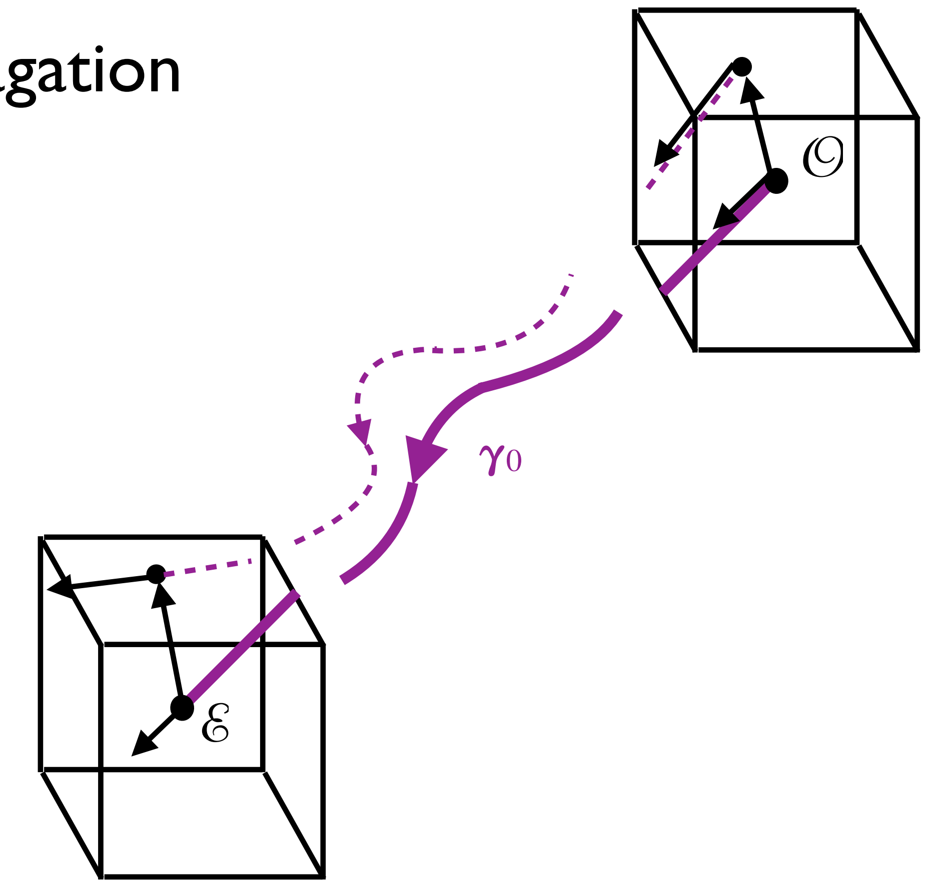


Summary

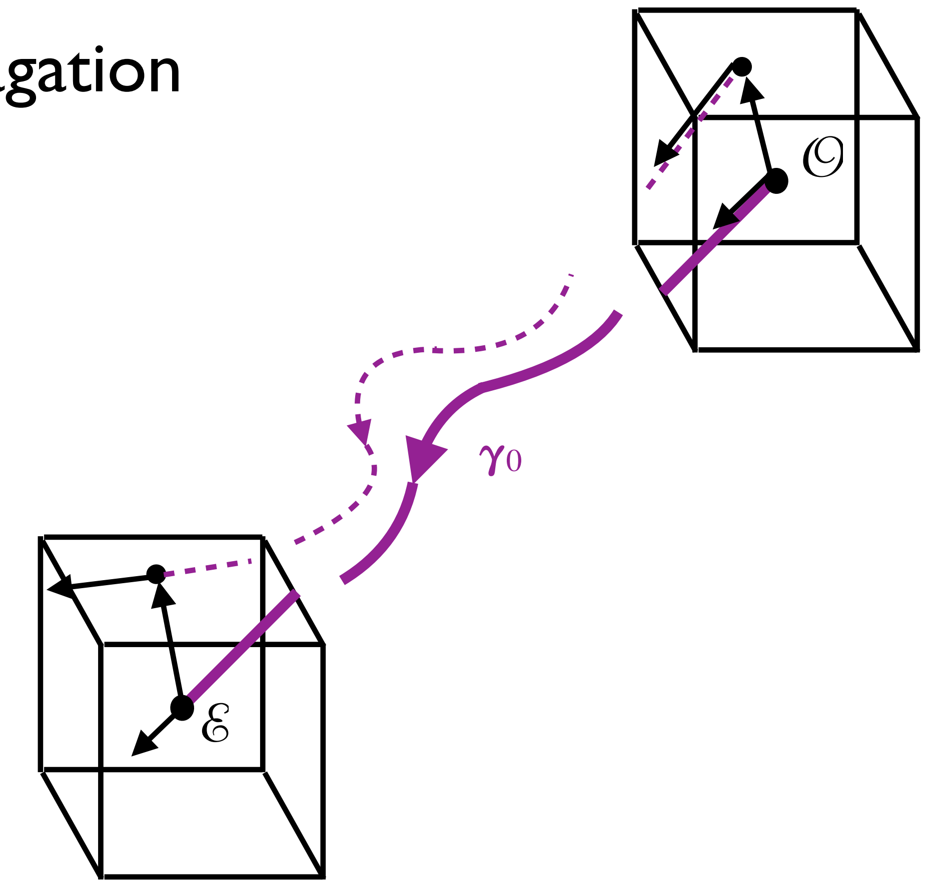
Covariant description of geometric optics and light propagation between small, distant regions

Valid in any spacetime (distant observer approximation)

Frame- (observer-) and coordinates-independent



Summary



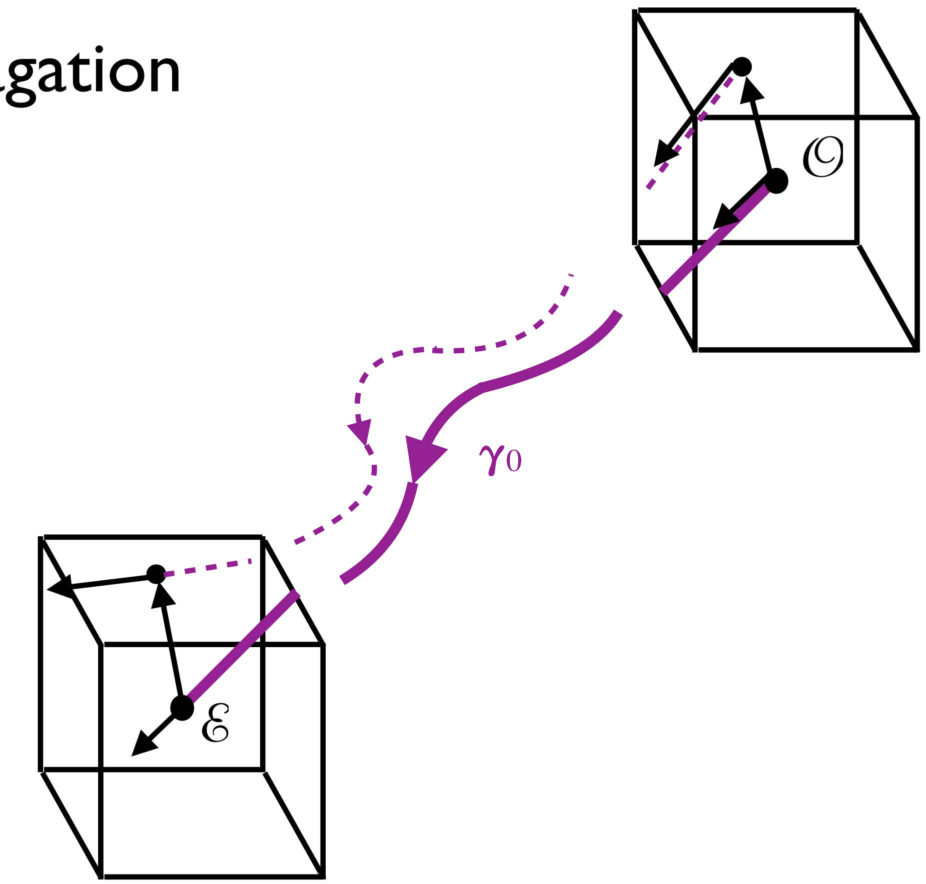
Covariant description of geometric optics and light propagation between small, distant regions

Valid in any spacetime (distant observer approximation)

Frame- (observer-) and coordinates-independent

Optical effects separated into propagation effects (GR) + local effects of motion (SR)

Summary



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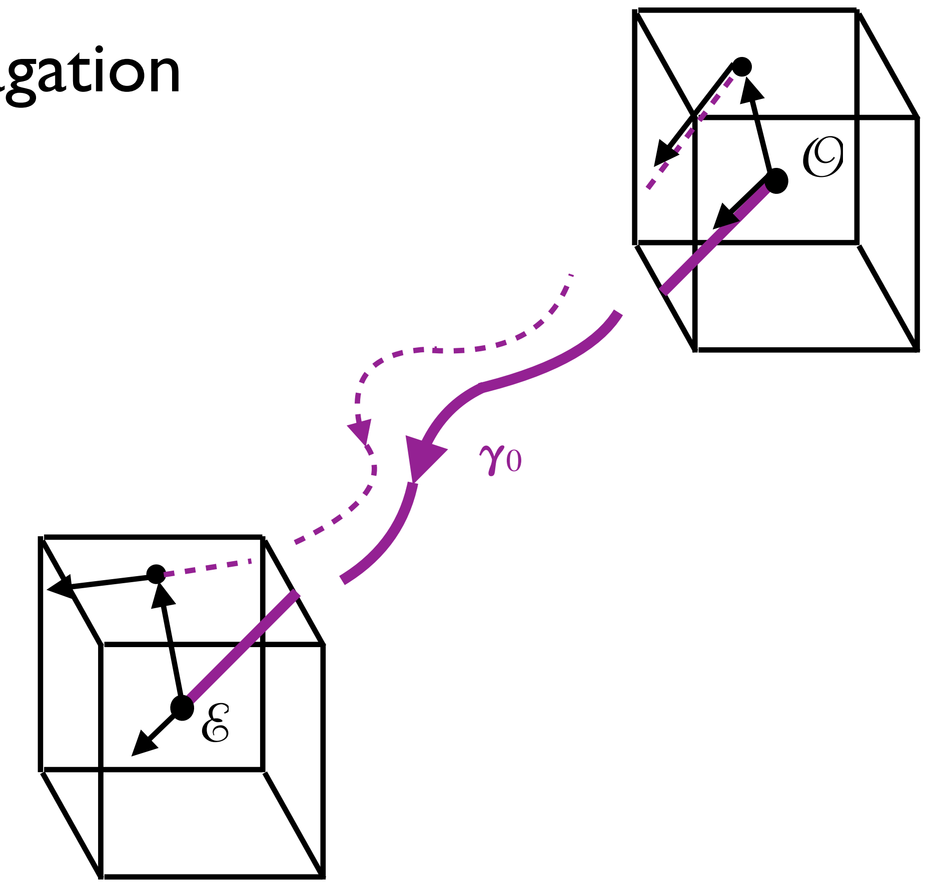
Optical effects separated into propagation effects (GR) + local effects of motion (SR)

GR effects encoded in 2 optical operators between appropriate quotient spaces

Summary



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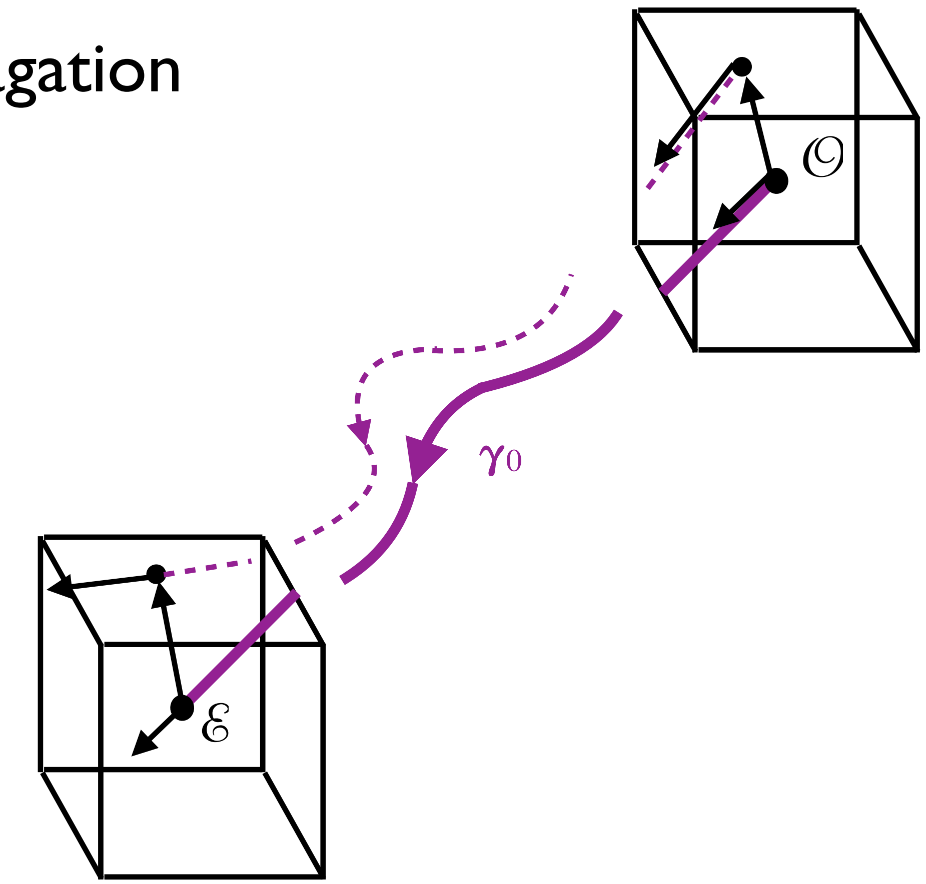

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Operators are linear functionals of the Riemann tensor along the line of sight

Image distortion, parallax, drift effects, redshift drift effects, Jacobi matrix drift, angular and luminosity distance drift

Applications + future research

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Cosmology: "real-time cosmology", comparison of positions and redshifts of distant objects after $\approx$10 ys

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Relativistic astrophysics - corrections to parallax

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Light propagation through small-scale inhomogeneities in the Universe

Applications + future research

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Thank you!