Killing spinor data on non-expanding horizons and the uniqueness of vacuum stationary black holes

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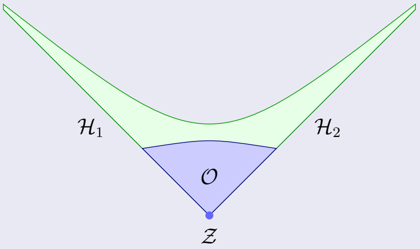
In this talk:

- I discuss how to use Rácz's holograph construction to analyse the existence of Killing spinors in the domain of dependence of the horizons of **distorted black holes**.
- I show how this analysis leads to conditions on the **bifurcation sphere** ensuring the existence of a Killing spinor.
- We obtain the most general 2-dimensional metric for which these conditions are satisfied.
- We show how to identify the free data in the holograph condition leading to a member of the **Kerr family** of solutions.

Outline

1. Rácz's holograph construction
2. Killing spinors in distorted black holes
3. Conclusions

Holograph construction for distorted black holes



Theorem (Rácz, 2014)

Suppose that $(\mathcal{M}, \boldsymbol{g})$ *is a vacuum spacetime with a vanishing Cosmological constant possessing a pair of null hypersurfaces* $\mathcal{H}_1$ *and* $\mathcal{H}_2$ *generated by* **expansion and shear-free** *geodesically complete null hypersurfaces* **intersecting** *on a 2-dimensional surface* $\mathcal{Z}$*. Then the metric* $\boldsymbol{g}$ *is uniquely determined (up to diffeomorphisms) on a neighbourhood* $\mathcal{O}$ *of* $\mathcal{Z}$ *contained in the domain of dependence* $D(\mathcal{H}_1 \cap \mathcal{H}_2)$ *once the metric* $\boldsymbol{\sigma}$ *on* $\mathcal{Z}$ *and a complex scalar are specified on* $\mathcal{Z}$*.*

Note:

- The spacetimes obtained from the previous construction are often called **distorted black holes**.

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Question:

- How to identify the **free data** in the holograph construction giving rise to the Kerr spacetime on $D(\mathcal{H}_1 \cap \mathcal{H}_2)$?

Killing spinors

Remark:

- A key element of our analysis is the notion of a (valence 2) **Killing spinor**.

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In a nutshell:

- A (valence 2) Killing spinor $\kappa_{AB} = \kappa_{(AB)}$ satisfies

$$H_{A'BC} \equiv \nabla_{A'(A}\kappa_{BC)} = 0.$$

- In terms of a normalised spin dyad $\{o^A, \iota^A\}$, κ_{AB} is described by 3 (complex) components

$$\kappa_0, \qquad \kappa_1, \qquad \kappa_2.$$

- The spinor

$$\xi_{AA'} \equiv \nabla^B{}_{A'}\kappa_{AB}$$

  satisfies the **Killing equation**

$$S_{AA'BB'} \equiv \nabla_{AA'}\xi_{BB'} + \nabla_{BB'}\xi_{AA'} = 0.$$

- The spinor $\xi_{AA'}$ is, in general, **not Hermitian**.

A characterisation of the Kerr spacetime

Theorem (Mars 1999; MJ Cole & JAVK 2016)

Let $(\mathcal{M}, \boldsymbol{g})$ *denote a smooth vacuum spacetime endowed with a (non-degenerate)* **Killing spinor** κ_{AB} *such that*

$$\xi_{AA'} \equiv \nabla^{B}{}_{A'}\kappa_{AB}$$

is **Hermitian**. *Then there exist two complex constants* $\mathfrak{l}$ *and* $\mathfrak{c}$ *such that*

$$\mathcal{H}^2 = -\mathfrak{l}(\mathfrak{c} - \chi)^4,$$

where χ *is the* **Ernst form** *and* $\mathcal{H}^2$ *is the* **norm of the Killing form**. *If, in addition,* $\mathfrak{c} = 1$ *and* $\mathfrak{l}$ *is real positive, then* $(\mathcal{M}, \boldsymbol{g})$ *is* **locally isometric to the Kerr spacetime**.

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Petrov type

- If $(\mathcal{M}, \boldsymbol{g})$ is endowed with a Killing spinor then

$$\Psi_{Q(ABC}\kappa_{D)}{}^{Q} = 0,$$

so that the Petrov type is D or more special.

Outline

Killing spinor characteristic initial data

Question:

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Killing spinor characteristic initial data

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- How to encode, in terms of characteristic data, the existence of a Killing spinor on $D(\mathcal{H}_1 \cup \mathcal{H}_2)$?

Strategy:

- Consider an **Killing spinor candidate** satisfying the equation

$$\square\kappa_{AB} + \Psi_{ABCD}\kappa^{CD} = 0,$$

such that

$$H_{A'ABC} = 0, \qquad S_{AA'BB'} = 0, \qquad \text{on} \quad \mathcal{H}_1 \cup \mathcal{H}_2.$$

- These conditions ensure that

$$H_{A'ABC} = 0, \qquad S_{AA'BB'} = 0, \qquad \text{on} \quad D(\mathcal{H}_1 \cup \mathcal{H}_2).$$

- Use a decomposition in terms of an **adapted spin dyad** to rewite the constraints on the spinor initial data in terms of conditions at $\mathcal{Z}$.

Conditions on the bifurcation sphere

Theorem

Let $(\mathcal{M}, \boldsymbol{g})$ *be a vacuum spacetime obtained through the* **holograph construction**. *Given a spin basis on* $\mathcal{Z} \approx \mathbb{S}^2$, *assume there exists a constant* $\mathfrak{K} \in \mathbb{C}$ *such that the conditions*

$$\kappa_0 = 0, \qquad \kappa_2 = 0,$$
$$\eth^2 \kappa_1 = \bar{\eth}^2 \kappa_1 = 0,$$
$$\kappa_1^3 \Psi_2 = \mathfrak{K}$$

hold on $\mathcal{Z}$. *Then* $D(\mathcal{H}_1 \cap \mathcal{H}_2)$ *is endowed with a Killing spinor. Moreover,* $\mathcal{Z}$ *is endowed with an* **axially symmetric Killing vector**.

Axial symmetry of the bifurcation sphere

Argument in a nutshell:

- On $\mathcal{Z}$ the Killing vector $\xi_{AA'}$ becomes **intrinsic** to the bifurcation sphere.
- The Killing equation on $\mathcal{Z}$ reads

$$\eth\xi_{01'} = 0, \qquad \bar{\eth}\xi_{10'} = 0, \qquad \eth\xi_{10'} + \bar{\eth}\xi_{01'} = 0.$$

- The above conditions are automatically satisfied if

$$\eth^2\kappa_1 = \bar{\eth}^2\kappa_1 = 0.$$

- A results by Wald (1990) then forces $\xi_{AA'}$ to be an (possibly complex) axial Killing vector on $\mathcal{Z}$.
- One then can show that the real and imaginary parts are **proportional**.

The metric at the bifurcation sphere

Proposition

There exists a **5- (real) parameter family** of smooth **axially symmetric 2-metrics** on $\mathcal{Z} \approx \mathbb{S}^2$ such that κ_1 is a solution to the constraints

$$\eth^2 \kappa_1 = 0, \qquad \bar{\eth}^2 \kappa_1 = 0$$

and such that the curvature condition

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also holds.

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Corollary

By resorting to the holograph construction, it follows there exists a **5-parameter family of distorted black hole solutions** endowed **with a Killing spinor**.

Identifying the Kerr spacetime

Observation (following Lewandowski et al, 2018):

- Using coordinates such that the **axially symmetric metric $\boldsymbol{\sigma}$** takes the form

$$\boldsymbol{\sigma} = R^2\left(Q^{-2}(\psi)d\psi^2 + Q^2(\psi)d\varphi^2\right),$$

one can solve the constraints on κ_1 to obtain

$$\kappa_1 = \mathfrak{c}\psi + \mathfrak{b}, \qquad \mathfrak{b},\ \mathfrak{c} \in \mathbb{C}.$$

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Proposition

The members of the family of spacetimes associated to the above solution to the constraints on $\mathcal{Z}$ which are **isometric to a member of the Kerr family** of metrics are characterised by the conditions

$$\mathfrak{c},\ \mathfrak{K} \in \mathbb{R}, \qquad \mathfrak{c} \in \mathbb{C} \setminus \mathbb{R}.$$

These conditions fix, in particular, the value of Ψ_2 on $\mathcal{Z}$.

Identifying the Kerr spacetime —strategy

Strategy of the proof:

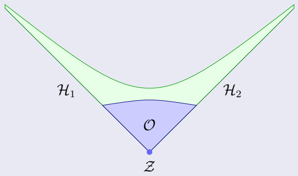
- To make use of the characterisation of the Kerr family in terms of Killing spinors impose constraints on $\mathcal{H}_1 \cup \mathcal{H}_2$ ensuring that $\xi_{AA'}$ is Hermitian.
- Reformulate these constraints as conditions on $\mathcal{Z}$ to obtain

$$\kappa_1 + \bar{\kappa}_1 = \text{constant}, \qquad \eth\bar{\eth}\kappa_1 + 2\Psi_2\kappa_1 \quad \text{is real.}$$

- The above conditions restrict the possible values of the constant $\mathfrak{c}$ to $\mathrm{i}c$.
- Final restrictions on $\mathfrak{K}$ and $\mathfrak{b}$ are obtained from Mars's Kerr characterisation result.

Outline

Concluding remarks



In this talk:

- A systematic **analysis of distorted black holes** endowed with Killing spinors.
- Used the existence of a Killing spinor to **identify the family of Kerr spacetimes**.
- Formulated the constraints giving rise to the Kerr spacetime in terms of **conditions on the bifurcation sphere** $\mathcal{Z}$.
- The analysis brings to the fore **the role of asymptotic flatness** in the uniqueness of stationary black holes.
- The analysis can be generalised in various obvious directions (Λ, electrovacuum, higher dimensions...)

Killing spinor data on distorted black hole horizons and the uniqueness of stationary vacuum black holes

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Abstract

We make use of the black hole holograph construction of [I. Rácz, Stationary black holes as holographs, Class. Quantum Grav. 31, 035006 (2014)] to analyse the existence of Killing spinors in the domain of dependence of the horizons of distorted black holes. In particular, we provide conditions on the bifurcation sphere ensuring the existence of a Killing spinor. These conditions can be understood as restrictions on the curvature of the bifurcation sphere and ensure the existence of an axial Killing vector on the 2-surface. We obtain the most general 2-dimensional metric on the bifurcation sphere for which these curvature conditions are satisfied. Remarkably, these conditions are found to be so restrictive that, in the considered particular case, the free data on the bifurcation surface (determining a distorted black hole spacetime) is completely determined by them. In addition, we formulate further conditions on the bifurcation sphere ensuring that the Killing vector associated to the Killing spinor is Hermitian. Once the existence of a Hermitian Killing vector is guaranteed, one can use a characterisation of the Kerr spacetime due to Mars to identify the particular subfamily of 2-metrics giving rise to a member of the Kerr family in the black hole holograph construction. Our analysis sheds light on the role of asymptotic flatness and curvature conditions on the bifurcation sphere in the context of the problem of uniqueness of stationary black holes. The Petrov type of the considered distorted black hole spacetimes is also determined.

Contents

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