On perturbing the domain of certain generators

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Joint work with Klaus-Jochen Engel, University of L'Aquila

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Outline

Objects of interest

First and second order differential operators on $\mathrm{L}^p([0,1],\mathbb{C}^m)$

$$G_1 := c(\bullet)\cdot \tfrac{d}{ds} \quad \text{and} \quad G_2 := a(\bullet)\cdot \tfrac{d^2}{ds^2}\,,$$

where $c(s), a(s) \in M_m(\mathbb{C})$, $s \in [0,1]$, are diagonal matrices and

$$D(G_1) := \left\{ f \in \mathrm{W}^{1,p}([0,1],\mathbb{C}^m) \mid \Phi f = 0 \right\}$$
$$D(G_2) := \left\{ f \in \mathrm{W}^{2,p}([0,1],\mathbb{C}^m) \mid \Phi_0 f = 0, \Phi_1(f' + Bf) = 0 \right\}$$

where $\Phi, \Phi_0, \Phi_1 \colon \mathrm{L}^p([0,1],\mathbb{C}^m) \to \mathbb{C}^m$ are "boundary" functionals and B a bounded linear operator on $\mathrm{L}^p([0,1],\mathbb{C}^m)$.

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Goal

Give conditions implying that G_1 and G_2 generate a C_0-semigroup.

Abstract results

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Greiner (1987), Salomon (1987), Weiss (1994), Staffans (2005), **Adler-Bombieri-Engel (2014)**, Hadd-Manzo-Rhandi (2015), . . .

Abstract results

Lemma (Greiner, 1987)

Let L be surjective. Then for each $\lambda \in \rho(A)$ the operator $L|_{\ker(\lambda - A_m)}$ is invertible and $L_\lambda := (L|_{\ker(\lambda - A_m)})^{-1} : \partial X \to \ker(\lambda - A_m) \subseteq X$ is bounded.

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Corollary

$L_A := (\lambda - A_{-1})L_\lambda \in \mathcal{L}(\partial X, X_{-1})$ *is independent of* $\lambda \in \rho(A)$ *and*

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Recall

The extrapolated space X_{-1} is the completion of X in the norm $\|x\|_{-1} := \|R(\lambda, A)x\|$, $T_{-1}(t) \in \mathcal{L}(X_{-1})$ is the unique bounded extension of $T(t)$ and A_{-1} its generator with $D(A_{-1}) = X$.

Theorem (Adler-Bombieri-Engel, 2014)

Assume that there exist $1 \le p < +\infty$, $t_0 > 0$ *and* $M \ge 0$ *such that*

(i) $\displaystyle\int_0^{t_0} T_{-1}(t_0 - s) L_A v(s)\, ds \in X$ *for all* $v \in \mathrm{L}^p([0,t_0],\partial X)$,

(ii) $\displaystyle\int_0^{t_0} \left\| \Phi\, T(s)x \right\|_{\partial X}^p ds \le M \cdot \|x\|_X^p$ *for all* $x \in D(A)$,

(iii) $\displaystyle\int_0^{t_0} \left\| \Phi \int_0^r T_{-1}(r-s) L_A v(s)\, ds \right\|_{\partial X}^p dr \le M \cdot \|v\|_p^p$

for all $v \in \mathrm{W}_0^{2,p}([0,t_0],\partial X)$,

(iv) $\mathcal{Q}_{t_0}$ *is invertible, where* $\mathcal{Q}_{t_0} \in \mathcal{L}\big(\mathrm{L}^p([0,t_0],\partial X)\big)$ *is given by*

$$(\mathcal{Q}_{t_0} v)(\bullet) = \Phi \int_0^{\bullet} T_{-1}(\bullet - s) L_A v(s)\, ds \text{ for all } v \in \mathrm{W}_0^{2,p}([0,t_0],\partial X).$$

Then $(G, D(G))$ *generates a* C_0*-semigroup on* X.

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Remark

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Lemma (Engel-KF, 2018)

Assumption (i) in previous Theorem is equivalent to: there exists $t_0 > 0$ and a strongly continuous family $(\mathcal{B}_t)_{t\in[0,t_0]} \subset \mathcal{L}(\mathrm{L}^p([0,t_0],\partial X),X)$ such that for every $u \in \mathrm{W}_0^{2,p}([0,t_0],\partial X)$ the function $x : [0,t_0] \to X$, $x(t) := \mathcal{B}_t u$ is a classical solution of the boundary control problem

$$\begin{cases} \dot{x}(t) = A_m x(t), & 0 \le t \le t_0, \\ Lx(t) = u(t), & 0 \le t \le t_0, \\ x(0) = 0. \end{cases}$$

In this case

$$\mathcal{B}_t u = \int_0^t T_{-1}(t-s) L_A u(s)\, ds \qquad \text{for } u \in \mathrm{L}^p([0,t_0],\partial X).$$

Back to our differential operators

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Assumptions and notations

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- For all $t \in [0, t_0]$ and $u \in \mathrm{W}_0^{1,p}([0, t_0], \mathbb{C}^m)$ define

$$\mathcal{R}_{t_0} u(t) := \bar{\Phi} \left(P_+ \hat{u} \left(t - \frac{1 - \bullet}{|\bar{c}|} \right) - P_- \hat{u} \left(\frac{\bullet}{|\bar{c}|} \right) \right)$$

where $\hat{u}$ is the extension of a function u to $\mathbb{R}$ by the value 0.

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Corollary

Let $V_0, V_1 \in \mathrm{M}_m(\mathbb{C})$. Operator

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$$D(G_1) = \big\{ f \in \mathrm{W}^{1,p}([0,1], \mathbb{C}^m) \mid V_0 f(0) = V_1 f(1) \big\}$$

is a generator if

$$\det\big(V_1 P_+ + V_0 P_-\big) \neq 0.$$

Example ($m = 2$)

Conditions in the domain:

$$f_1(0) = f_2(1), \quad f_2(0) = f_1(1).$$

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Then

$$V_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad V_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and G_1 is a generator if either both $c_1(\bullet)$ and $c_2(\bullet)$ are strictly positive or both are strictly negative.

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$$G_2 = a(\bullet) \cdot \tfrac{d^2}{ds^2}$$
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- $a(\bullet)$ positive Lipshitz continuous
- For $t \in [0, t_0]$ and $u, v \in \mathrm{W}_0^{1,p}([0,t_0],\mathbb{C}^m)$ define $\mathcal{R}_{t_0}\binom{u}{v}(t)$ as

$$\Phi_0\left(\hat{u}\left(t + \tfrac{\bullet - 1}{\bar{a}}\right) - \hat{v}\left(t - \tfrac{\bullet}{\bar{a}}\right)\right) + \Phi_1\left(\hat{u}\left(t + \tfrac{\bullet - 1}{\bar{a}}\right) + \hat{v}\left(t - \tfrac{\bullet}{\bar{a}}\right)\right)$$

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Theorem (Engel-KF, 2018)

If there exists $t_0 > 0$ *such that the operator* $\mathcal{R}_{t_0}$ *is invertible, then* G_2 *generates a* C_0*-semigroup on* $\mathrm{L}^p([0,1],\mathbb{C}^m)$.

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Corollary

Take $k_0, k_1 \in \mathbb{N}$ satisfying $k_0 + k_1 = 2m$ and matrices $V_0, V_1 \in \mathrm{M}_{k_0\times m}(\mathbb{C})$, $W_0, W_1 \in \mathrm{M}_{k_1\times m}(\mathbb{C})$. If

$$\det\begin{pmatrix} V_1 & V_0 \\ W_1 \cdot a(1)^{-1/2} & -W_0 \cdot a(0)^{-1/2} \end{pmatrix} \neq 0,$$

then the operator G_2 with conditions in the domain

$$V_0 f(0) = V_1 f(1), \quad W_0 f'(0) = W_1 f'(1) + (Bf)(1)$$

generates a C_0-semigroup.

Back to our differential operators

Example ($a(\bullet) = Id,\ m = 1$)

$G_p,\ G_D \subset \frac{d^2}{ds^2}$ on $X := \mathrm{L}^p[0,1]$ with domains

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$$G_p: \ k_0 = k_1 = 1,\ V_1 = -1,\ V_0 = W_0 = W_1 = 1,$$
$$\det \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \neq 0 \quad \Rightarrow \quad G_P \text{ generator}$$

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$$G_D: \ k_0 = 2,\ k_1 = 0,\ V_0 = \binom{1}{0},\ V_1 = \binom{0}{1},$$
$$\det \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix} \right) \neq 0 \quad \Rightarrow \quad G_D \text{ generator}$$

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Example ($m = 1$, nonconstant coefficients)

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$$D(G_2) := \left\{ f \in \mathrm{W}^{2,p}[0,1] \mid f(0) + f(1) = 0,\ f'(0) - f'(1) = 0 \right\}.$$

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Taking $k_0 = k_1 = 1,\ V_0 = W_0 = W_1 = V_1 = 1$ we obtain

$$\det \begin{pmatrix} V_1 & V_0 \\ \bar{W}_1 & \bar{W}_0 \end{pmatrix} \neq 0 \quad \Longleftrightarrow \quad a(0) \neq a(1) \quad \Longleftrightarrow \quad G_2 \text{ generator}$$

Invitation

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