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Asymptotics of the Lebowitz–Rubinow–Rotenberg model of cell populations development

We consider a mathematical model of cell populations dynamics proposed by J. Lebowitz and S. Rubinow, and analysed by M. Rotenberg. In the model each cell is characterized by her maturity and speed of maturation. The growth of cell populations is described by a partial differential equation with a boundary condition. More precisely growth of the cells' population density is governed by the partial differential equation

$$\frac{\partial f}{\partial t} = -v\frac{\partial f}{\partial x},$$

where $f = f(x, v, t)$ with $t \geq 0$ is the cells' density at (x, v) at time t. In this model a cell starts maturing at $x = 0$ and divides reaching $x = 1$, and the boundary condition

$$vf(0, v, t) = p\int_V wk(w, v)f(1, w, t)dw$$

describes the reproduction rule. Here k satisfies

$$\int_V k(w, v)dv = 1$$

for any $w \in V$, and $V \ni v \mapsto k(w, v)$ is the probability density of the daughter's maturation velocity conditional on w being the velocity of the mother. Furthermore, it is assumed that $p \geq 0$ is the average number of viable daughters per mitosis.

By applying the Lord Kelvin method of images we give a new proof that the model is well posed. A semi-explicit formula for the semigroup related to the model obtained by the method of images allows also two types of new results. First of all, we give growth order estimates for the semigroup, applicable also in the case of decaying populations. Secondly, we study asymptotic behavior of the semigroup in the case of approximately constant population size. More specifically, we formulate conditions for the asymptotic stability of the semigroup in the case in which the average number of viable daughters per mitosis equals one.