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On a model of the ideal heat exchanger and its relation to the telegrapher's equations

We examine a model of the ideal heat exchanger. In the ideal heat exchanger we have two one-dimensional objects (bars or columns of liquid in pipes) of lengths a and b, respectively. The linear heat capacities of the objects are p and q, respectively. The heat is not transferred along the objects. It is only transferred between the objects and the linear thermal conductivity is α (the case $\alpha \to \infty$ is especially interesting). The objects are moving and one passes along another with the constant velocity 1. Let $\widetilde{f} : [0, a] \to \mathbb{R}$ and $\widetilde{g} : [0, b] \to \mathbb{R}$ be continuous functions representing the initial temperature distribution along the objects. We examine the time evolution of the temperature distribution. Our model is described by the following system of equations:

$$\frac{\partial f_\alpha(x,t)}{\partial t} = \begin{cases} \frac{\alpha}{p} \cdot (g_\alpha(t-x,t) - f_\alpha(x,t)) & \text{if } x \in [0,a] \text{ and } t-x \in (0,b) \\ 0 & \text{if } x \in [0,a] \text{ and } t-x \notin [0,b] \end{cases}$$

$$\frac{\partial g_\alpha(x,t)}{\partial t} = \begin{cases} \frac{\alpha}{q} \cdot (f_\alpha(t-x,t) - g_\alpha(x,t)) & \text{if } x \in [0,b] \text{ and } t-x \in (0,a) \\ 0 & \text{if } x \in [0,b] \text{ and } t-x \notin [0,a] \end{cases}$$

where $f_\alpha : [0, a] \times \mathbb{R}_+ \to \mathbb{R}$ and $g_\alpha : [0, b] \times \mathbb{R}_+ \to \mathbb{R}$ and the initial conditions are $f_\alpha(x, 0) = \widetilde{f}(x)$ and $g_\alpha(x, 0) = \widetilde{g}(x)$. The above system is not autonomous and it does not determine any semigroup. However, it is closely related to the telegrapher's equations (which can be seen after changing the variables), but its behaviour is different. For example, despite the continuity of $\widetilde{f}$ and $\widetilde{g}$, the limits of f_α and g_α for $\alpha \to \infty$ do not need to be continuous.