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Bi-Laplacians on graphs and networks

We study the differential operator $A = \frac{d^4}{dx^4}$ acting on a connected network $\mathcal{G}$ along with $\mathcal{L}^2$, the square of the discrete Laplacian acting on a connected combinatorial graph G. For both operators we discuss self-adjointness issues, well-posedness of the associated linear parabolic problems

$$\frac{df}{dt} = -\mathcal{L}^2 f, \qquad \frac{\partial u}{\partial t} = -Au,$$

and extrapolation of the generated semigroups to consistent families of semigroups on $L^p(\mathcal{G})$ or $\ell^p(\mathsf{V})$, respectively, for $1 \leq p \leq \infty$. Our most surprising finding is that, upon allowing the system enough time to reach diffusive regime, the parabolic equations driven by $-A$ may display Markovian features: analogous results seem to be unknown even in the classical case of domains. This kind of analysis is based on a detailed study of bi-harmonic functions complemented by simple combinatorial arguments; we emphasize the role of boundary conditions in the vertices of the network, and in turn of the network's connectivity, in determining such Markovian properties. We elaborate on analogous issues for the discrete bi-Laplacian; a characterization of complete graphs in terms of the Markovian property of the semigroup generated by $-\mathcal{L}^2$ is also presented.

Joint work with Delio Mugnolo.