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Asymptotic stability of the evolutionary Boltzmann - type equations

Some problems of the mathematical physics can be written as differential equations for functions with values in the space of measures. The vector space of signed measures doesn't have good analytical properties. For example, this space with the Fortet-Mourier metric is not complete.

There are two methods to overcome this problem. First, we may replace the original equations by the adjoint ones on the space of continuous bounded functions. Secondly, we may restrict our equations to some complete convex subsets of the vector space of measures. This approach seems to be quite natural and it is related to the classical results concerning differential equations on convex subsets of Banach spaces (see [2]). The convex sets method in studying the Boltzmann equation was used in a series of papers (see for example : [1, 3, 5, 6, 7]).

The main purpose of our study is to show some application of the Kantorovich-Rubinstein maximum principle concerning the properties of probability metrics. Then we show that the Kantorovich-Rubinstein maximum principle combined with the LaSalle invariance principle used in the theory of dynamical systems allow us to find new sufficient conditions for the asymptotic stability of solutions of a same version of the nonlinear Boltzmann-type equation. This equation was stimulated by the problem of the stability of solutions of the following version of the Boltzmann equation (see [3])

$$\frac{d\psi}{dt} + \psi = P\,\psi \qquad \text{for} \qquad t \geq 0 \tag{1}$$

with the initial condition

$$\psi\,(0) = \psi_0, \tag{2}$$

where $\psi_0 \in \mathcal{M}_1(\mathbb{R}_+)$ and $\psi : \mathbb{R}_+ \to \mathcal{M}_{sig}(\mathbb{R}_+)$ is an unknown function. Moreover P is the collision operator acting on the space of probability measures.

We will discuss a equation drawn from the kinetic theory of gases which will be a generalized version of (1) (see [4]). We will assume that the collision operator P is a convex combination of N operators $P_1, ..., P_N$, where P_k for $k \geq 2$ describes the simultaneous collision of k particles and P_1 the influence of external forces.

The same equation which was discussed by Lasota [5]. This stability result was based on the technique of Zolotarev metrics. Lasota showed that the stationary solution is exponentially stable in the Zolotarev norm of order 2.

The basic idea of our method is to apply the Kantorovich-Rubinstein maximum principle combined with the LaSalle invariance principle. More precisely we will show that if our equation has a stationary measure $\mu_\star$ such that $\mathrm{supp}\mu_\star = \mathbb{R}_+$, then this measure is asymtotically stable with respect to the Kantorovich-Wasserstein metric.

The open problem related with characterization of the stationary measure $\mu_\star$ of the equation (1) will end the talk.

Refer to the positions in the reference list as follows: [1], [2].

References

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