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Hot spots of quantum graphs

Let $(e^{t\Delta_\Omega^N})_{t\geq 0}$ denote the semigroup associated with the Neumann Laplacian on a bounded Euclidean domain $\Omega \subset \mathbb{R}^d$. The Hot Spots Conjecture of J. Rauch asserts that for a "generic" initial condition u_0, if $x_t \in \overline{\Omega}$ is any point at which $u(t,x) := e^{t\Delta_\Omega^N} u_0(x)$ reaches is maximum (or minimum), then $x_t \to \partial\Omega$ as $t \to \infty$. In words, the hottest and coldest points of the body Ω should generically move towards its boundary for large times, if the insulation is perfect. In its most common formulation, this reduces to proving that maximum and minimum of the eigenfunction(s) associated with the smallest positive eigenvalue μ_2 of $-\Delta_\Omega^N$ are located on the boundary. This conjecture is not true in full generality [2] but is currently open, for example, for convex domains [1, 3].

In this talk we will examine the corresponding question on metric graphs: if μ_2 denotes the smallest positive eigenvalue of the Laplacian with standard (continuity and Kirchhoff) vertex conditions, we consider the possible distribution of maxima and minima of eigenfunctions associated with μ_2. Among other things, we give examples to show that the usual notion of "boundary" of a metric graph, namely the set of vertices of degree one, has limited relevance for determining the "hottest" and "coldest" parts of a graph.

This is based on ongoing joint work with Jonathan Rohleder (University of Stockholm), see also [4].

References

[1] R. Bañuelos and K. Burdzy, *On the hot spots conjecture of J. Rauch*, J. Funct. Anal. **164** (1999), 1–33.

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[3] C. Judge and S. Mondal, *Euclidean triangles have no hot spots*, preprint (2018), arXiv:1802.01800.

[4] J. Kennedy and J. Rohleder, *On the hot spots of quantum trees*, preprint (2018), arXiv:1808.02274.