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Fundamental solution for super-critical non-symmetric Lévy-type operators

In the talk I will present the results of the paper [4], which is a sequel to [2]. The aim is to construct the fundamental solution p^κ to the equation $\partial_t = \mathcal{L}^\kappa$, where under certain assumptions the operator \mathcal{L}^κ takes the form,

$$\mathcal{L}^\kappa f(x) := \int_{\mathbb{R}^d} (f(x+z) - f(x) - 1_{|z|<1} \langle z, \nabla f(x) \rangle) \kappa(x, z) J(z) dz.$$

In particular, $J: \mathbb{R}^d \rightarrow [0, \infty]$ is a Lévy density, i.e., $\int_{\mathbb{R}^d} (1 \wedge |x|^2) J(x) dx < \infty$. The function $\kappa(x, z)$ is assumed to be Borel measurable on $\mathbb{R}^d \times \mathbb{R}^d$ satisfying $0 < \kappa_0 \leq \kappa(x, z) \leq \kappa_1$, and $|\kappa(x, z) - \kappa(y, z)| \leq \kappa_2 |x - y|^\beta$ for some $\beta \in (0, 1)$.

We concentrate on the case when the order of the operator is positive and smaller or equal 1 (without excluding higher orders up to 2). The lack of the symmetry of the Lévy density $\kappa(x, z)J(z)$ in z variable may cause a non-zero *internal drift*, which reveals itself as a gradient term in the decomposition

$$\begin{aligned} \mathcal{L}^\kappa f(x) &= \int_{\mathbb{R}^d} (f(x+z) - f(x) - 1_{|z|<r} \langle z, \nabla f(x) \rangle) \kappa(x, z) J(z) dz \\ &\quad + \left(\int_{\mathbb{R}^d} z (1_{|z|<r} - 1_{|z|<1}) \kappa(x, z) J(z) dz \right) \cdot \nabla f(x). \end{aligned}$$

Our approach rests on imposing conditions on the expression

$$\int_{r \leq |z| < 1} z \kappa(x, z) J(z) dz.$$

We prove the uniqueness, estimates, regularity and other qualitative properties of p^κ . The result is new even for 1-stable Lévy measure $J(z) = |z|^{-d-1}$, cf. [3] and [1].

References

- [1] Zhen-Qing Chen and Xicheng Zhang, *Heat kernels for time-dependent non-symmetric stable-like operators*, preprint 2017, arXiv:1709.04614.
- [2] Tomasz Grzywny and Karol Szczypkowski, *Heat kernels of non-symmetric Lévy-type operators*, preprint 2018, arXiv:1804.01313.
- [3] P. Jin, *Heat kernel estimates for non-symmetric stable-like processes*, preprint 2017, arXiv:1709.02836.
- [4] Karol Szczypkowski, *Fundamental solution for super-critical non-symmetric Lévy-type operators*, preprint 2018, arXiv:1807.04257.