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Uni-asymptotic linear systems

We consider abstract systems of the form $\{U_s\}_{s\in S}$, where $U_s : X \longrightarrow X$ are bounded linear operators in a norm space X, and S is an arbitrary set. Such a system can be treated, e.g., as formal form of a dynamical system, describing a certain real process as follows: S represents the set of "admissible moments of time", X — the set of "admissible states", and if the initial state of the process is $x \in X$, then $U_s x$ is the state of the process at the time moment $s \in S$.

Our goal is to analyze such systems $\{U_s\}_{s\in S}$, that all their non-trivial trajectories $\{U_s x\}_{s\in S}$ (with $x \neq 0$) "have the same norm-asymptotic behavior" — we call them *uni-asymptotic*. On the other hand, $\{U_s\}_{s\in S}$ is *tight*, when the operator norm and the minimal modulus of U_s "have the same asymptotic behavior".

We prove that uni-asymptoticity is equivalent to tightness if $\dim X < +\infty$. We also prove that the finite dimension is essential above. Some other conditions equivalent to uni-asymptoticity are provided, including asymptotic formulae for the operator norm and for the trajectories, expressed in terms of determinants $\det U_s$.