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Diffusion with nonlocal Dirichlet boundary conditions on unbounded domains.

In [1], we have considered second order strictly elliptic operators of the form

$$\mathcal{A}u = \sum_{i,j=1}^{d} a_{ij}\partial_i\partial_j u + \sum_{j=1}^{d} b_j\partial_j u$$

on a bounded, Dirichlet regular set $\Omega \subset \mathbb{R}^d$. These operators were complemented by nonlocal Dirichelet boundary conditions:

$$u(z) = \int_\Omega u(x)\mu(z, dx), \quad z \in \partial\Omega.$$

These boundary conditions have a clear probabilistic interpretation. When a particle, diffusing according to the operator $\mathcal{A}$, hits the boundary in a point z, it jumps instantly back to the interior. The position it jumps to is chosen according to the distribution $\mu(z, \cdot)$. In [1] we were able to prove that a realization of this operator generates a semigroup on the space $C(\overline{\Omega})$. It should be noted that this semigroup is an analytic semigroup, but*not* strongly continuous.

In this talk we will present an extension of this result where we allow Ω to be an unbounded set and the coefficients of the operator $\mathcal{A}$ to be unbounded near infinity. It should be noted that the semigroup generated by such an operator can no longer be expected to be analytic, so that the rich theory of analytic semigroups is no longer at our disposal to compensate for the lack of strong continuity. Instead, we present an approach based on approximation, where we approximate the unbounded domain Ω with bounded domains Ω_n. We will also discuss additional properties of the semigroup such as the strong Feller property and asymptotic behavior.

References

[1] Wolfgang Arendt, Stefan Kunkel, and Markus Kunze, *Diffusion with nonlocal boundary conditions*, J. Funct. Anal. **270** (2016), no. 7, 2483–2507. MR 3464048