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Relative operator bounds for positive operators in ordered Banach spaces and related topics

It is known that if $A : D(A) \subset X \to X$ is a semibounded (say bounded from above) self-adjoint operator in a Hilbert space X and if $S : D(A) \to X$ is A-bounded then the *relative A-bound* of S is equal to

$$\lim_{\lambda \to \infty} \left\| S(\lambda - A)^{-1} \right\|_{\mathcal{L}(X)}.$$

(This result which is still true when A is just α-dissipative *is no longer true outside the realm of Hilbert spaces.*) We show that this result remains true in real ordered Banach spaces X whose *duality map is uniformly monotone on the positive cone* (this covers L^p spaces with $p \geq 2$) if A generates a positive C_0-semigroup on X and S is positive, i.e. $S : D(A) \cap X_+ \to X_+$. This result turns out to be a consequence of *"contractivity on the positive cone"* of $I - C$ for certain positive contractions C. Related results on ergodic projections are also given.

Ref: Contractivity results in ordered spaces. Applications to relative operator bounds and projections with norm one. *Math. Nachr*, 290 (2016).