Natalia Manakova
South Ural State University

Optimal Control Problem for the Sobolev Type Equations

A lot of initial-boundary value problems for the equations and the systems of equations not resolved with respect to time derivative are considered in the framework of abstract Sobolev type equations

$$L\,\dot{x}= Mx + Bu + y, \quad \ker L \neq \{0\}, \tag{1}$$

and

$$L\,\dot{x} + Mx + N(x) = u, \quad \ker L \neq \{0\}, \tag{2}$$

that make up the vast field of non-classical equations of mathematical physics. The Cauchy problem

$$x(0) - x_0 = 0 \tag{3}$$

for degenerate equations (1) or (2) is unsolvable for arbitrary initial values. We consider the Showalter – Sidorov problem

$$L(x(0) - x_0) = 0 \tag{4}$$

for semi-linear equation (2) and the initial-final problem

$$P_{in}(x(0) - x_0) = 0, \quad P_{fin}(x(0) - x_0) = 0 \tag{5}$$

for linear equation (1), which is a natural generalization of the Cauchy problem.

Generally, the processes applied in mechanics, engineering and production are controllable, therefore, within respective applied problems it is usually essential to control the external actions efficient enough to achieve required results in such processes. Despite the fact that the research field of optimal control problems for distributed systems is rather large, the solutions control matters for confluent semi-linear and linear equations, unresolved for derivative with time, are studied insufficiently. The optimal control problem

$$J(x, u) \to \inf \tag{6}$$

for a Sobolev type equation was first considered by G.A. Sviridyuk and A.A. Efremov. This research provided the basis for a branch of optimal control studies referring to linear and semi-linear Sobolev type equations.

This talk introduces a sufficient conditions for the existence of a solution of the optimal control problems for linear and semi-linear Sobolev type equations. The theory is based on the phase space method. The developed scheme of a numerical method allows one to find an approximate solution to optimal control problems for considered models. On the basis of abstract results the existence of optimal control of processes of filtration and deformation are obtained.