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Discrete Fragmentation Equations

The process of fragmentation arises in many physical situations, including depolymerisation, droplet break-up and rock fracture. In some cases, it is possible to model processes of this type using a discrete size variable. For example, if the fragmenting objects, such as polymers, can be interpreted as clusters composed of identical fundamental units (e.g. monomers), then clusters of size n (n-mers) are those which consist of n fundamental units. Under suitable assumptions, the evolution of the number concentration of clusters of all positive integer sizes can be described by an infinite linear system of ordinary differential equations. In the first part of this talk, it will be shown that the associated initial-value problem for this infinite-dimensional system can be expressed as an abstract Cauchy problem, posed in a physically relevant Banach space. Routine application of a result on stochastic semigroups by Thieme and Voigt then leads to the existence and uniqueness of classical solutions.

Although it is usual to assume that the fragmentation process conserves the total mass of all clusters, there are examples, such as bond annihilation, in which mass loss can arise in a natural manner. The second part of the talk is based on joint work [1] with Louise Smith, Matthias Langer and Adam McBride (University of Strathclyde), and focusses on a simple model of random bond annihilation for which it is possible to establish that the associated fragmentation semigroup is analytic. This model will also be used to highlight the problem of non-uniqueness of pointwise solutions to fragmentation systems. In particular, it will be shown that there is an explicit, non-trivial, solution that satisfies homogeneneous initial conditions in a pointwise manner. This apparent paradox will be explained in a satisfactory manner by using the theory of Sobolev towers.

If time permits, some brief comments will also be made on the benefits of having an analytic fragmentation semigroup when dealing with related, semilinear coagulation-fragmentation equations.

References

[1] L. Smith, W. Lamb, M. Langer, and A. McBride, *Discrete fragmentation with mass loss*, Journal of Evolution Equations **12** (2012), no. 1, 181–201.