Title *My Love affair with Thin Triangles and Fat Annuli*

Abstract Each plane domain Ω (with at least two boundary points) supports both a hyperbolic distance h and a quasihyperbolic distance k. In general, the metric spaces (Ω, h) and (Ω, k) may be quite different. Nonetheless, the underlying geometries are strikingly similar, and we discuss recent results that illustrate this phenomenon.

For example, the metric spaces are always conformally equivalent, and it is well known that they are bi-Lipschitz equivalent if and only if $\hat{\mathbb{C}} \setminus \Omega$ is uniformly perfect. In fact, there is rigidity in that the two metric spaces are either bi-Lipschitz equivalent (so quite similar) or not quasisymmetrically equivalent (so quite different).

However, the two metric spaces, which are both geodesic, always have exactly the same quasi-geodesics (and this is quantitative), and one space is Gromov hyperbolic if and only if the other is too; and when so, their Gromov boundaries are always quasisymmetrically equivalent (i.e., the canonical conformal gauges are quasisymmetrically equivalent). Moreover, for a large class of domains (including all finitely connected domains and many infinitely connected ones too, even without uniformly perfect complements), the two metric spaces are quasiisometrically equivalent.