AM–MODULUS AND CODIMENSION ONE IN METRIC MEASURE SPACES
ABSTRACT

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In a metric measure space (X, d, μ) the AM– modulus of a path family Γ is defined as

$$AM(\Gamma) = \inf\Big\{\liminf_i \int_X \rho_i \, d\mu : \ \rho_i \geq 0 \text{ Borel}, \ \liminf_i \int_\gamma \rho_i \, ds \geq 1, \ \forall\, \gamma \in \Gamma\Big\}.$$

It is shown that

$$co\mathcal{H}^1(E) \approx AM(\Gamma(E))$$

for all Suslin sets E in X. Here $co\mathcal{H}^1$ stands for the Hausdorff measure of codimension one in X and $\Gamma(E)$ is the family of all non–constant paths which meet the set $E \subset X$. The proof uses the perimeter concept and a capacity different from the standard capacity based on the $N^{1,1}(X)$–functions. The result is new in $\mathbb{R}^n$.

The result gives a new characterization of sets of finite perimeter in X as well as new information on measure theoretic properties of sets in X.

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