JN_p – A GENERALIZATION OF BMO

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ABSTRACT. In 1961, John and Nirenberg introduced functions of bounded mean oscillation (BMO) and proved that they are exponentially integrable. In the same paper, they also considered a class of larger function spaces, which are now known as JN_p and proved that these functions lie in weak L^p. The space BMO can be seen as the limit of JN_p as $p \to \infty$. It is rather easy to see that JN_p contains L^p and that all weak L^p functions are not in JN_p. It turned out to be more difficult to find a function that is in $\mathrm{JN}_p \setminus L^p$, i.e. it was not known, whether JN_p is just a new name for L^p. One reason is that all monotone JN_p functions are in L^p.

We provide an example of a function in $\mathrm{JN}_p \setminus L^p$. We also characterize JN_p as the dual of a new space defined in terms of atomic decomposition. This is joint work with G. Dafni, T. Hytönen, and H. Yue.