Spectral multipliers and wave equation for sub-Laplacians

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Mihlin–Hörmander theorem gives the sharp Sobolev order $n/2$ for a spectral multiplier of the Laplacian to define a bounded operator on $L^p(\mathbb{R}^n)$ for all $p \in (1, \infty)$. We study the same type of statements for sub-Laplacians, which are sub-elliptic operators defined on sub-Riemannian manifolds.

Although a Mihlin–Hörmander-type theorem in Carnot groups is known, the sharp Sobolev order is still unknown. It is conjectured to be $n/2$, where n is the topological dimension.

We have proven that in no sub-Riemannian manifold the sharp Sobolev order can be lower than $n/2$, where n is the topological dimension. For the proof, we construct a partial representation of the sub-Riemannian half-wave propagator as a Fourier integral operator. For such Fourier integral operator, the critical points of the phase function are determined by the sub-Riemannian exponential map.

This is a joint work with Alessio Martini and Detlef Müller.