THE DIRICHLET PRINCIPLE FOR INNER VARIATIONS

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We are largely concerned with the Dirichlet energy in the complex plane. Historically, the existence of the energy-minimal solutions (with prescribed boundary values) was based on physical interpretations of the problem. This was taken for granted (until Karl Weierstrass constructed a counter-example) by numerous eminent mathematicians, including Bernhard Riemann who coined the term *Dirichlet's Principle*. It asserts that:

> A function is harmonic (satisfies the Laplace equation) if and only if its *outer variations* increase the energy.

When an energy-minimal mapping is subjected to additional topological constraints; for example, to be a homeomorphism (due to the law of non-interpenetration of matter in Nonlinear Elasticity), the outer variations are unavailable. This is where the *inner variation* and the corresponding variational *Hopf-Laplace equation* come in. An analogue of the Dirichlet Principle for the solutions (called *Hopf harmonics*) reads as:

> A function satisfies Hopf-Laplace equation if and only if its *inner variations* increase the energy.

However, the proof works rather differently.

As we seek greater knowledge about inner-variations for general energy functionals, the Dirichlet type principles become ever more basic. I will summarize our recent advances with ***Jani Onninen***.

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