FRACTIONAL RECTIFIABILITY

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Abstract. The traveling salesman problem, one of the most renowned problems in computer science, asks for the shortest path that passes through a given finite set of points in R^n. More generally, given a (finite or infinite) set $E \subset R^n$, when is it possible to construct a Lipschitz map from the unit interval into R^n that contains E in its image? While Lipschitz 1-rectifiability has been thoroughly studied and completely characterized, very little is known about the Hölder case. When $s > 1$, $(1/s)$-Hölder curves are more exotic objects than rectifiable curves and include fractal curves and space-filling curves as basic examples.

In this talk we discuss several advances towards Hölder rectifiability including an extension of Peter Jones' traveling salesman construction, which provides a sufficient condition for E to be contained in a $(1/s)$-Hölder curve for $s \geq 1$. We also discuss the Hölder geometry of curves generated by iterated function systems (IFS) in a complete metric space. This improves a theorem of Remes from 1998, which previously established that connected self-similar sets in Euclidean space that satisfy the open set condition are Hölder curves. This talk is based on joint works with Matthew Badger and Lisa Naples.

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