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One-parameter families of hyperbolic iterated function systems on the line

In the last four decades, considerable attention has been paid to studying self-similar measures' dimensions and absolute continuity. We know much less about these questions if we replace the self-similar iterated function system (IFS) with a hyperbolic one. After introducing this topic, we learn a new technique (which is from a recent paper joint with Bárány, Solomyak, and Śpiewak) that extends the usual transversality method, which is essential for studying one-parameter families of hyperbolic IFSs.

More precisely, a hyperbolic IFS on the line is a finite list of strictly contracting smooth self-mappings of a compact interval into itself such that the derivatives are uniformly separated from zero. If the cylinders are well separated, then the dimension of the attractor is the root of the appropriate pressure formula. However, suppose we do not assume any separation conditions. In that case, we can hope to get results about the dimension of the attractor only if we consider a one-parameter family of such hyperbolic IFSs which satisfy the so-called transversality condition. In this case, we can determine the dimension of the attractor for typical parameters. To better understand these systems, we need to study the geometric and dimensional properties of the images under the natural projections of measures on the symbolic space. However, when we study a one-parameter family of hyperbolic IFSs, the significance of a measure depends on the parameter. Hence, for different parameters, different measures are the most relevant. So, we need to consider the (parameter-dependent) natural projection of parameter-dependent measures. However, with the standard transversality method, we can handle only the one-parameter family of measures on the line that we obtain by projecting from the symbolic space the same measure for all parameters. The novelty of the method I would like to present in the series of my talks is an extension of the transversality technique. Using this new method, we can study the dimension and absolute continuity of a family of measures on the line, which family is obtained by projecting with the (parameter-dependent) natural projection of parameter-dependent measures. Using this technique, we obtain that the Gibbs measures for a family of Hölder continuous potentials with Hölder continuous dependence on the parameter satisfying the transversality condition, the projected measure is absolutely continuous for Lebesgue a.e. parameter such that the ratio of entropy over the Lyapunov exponent is strictly greater than 1. As another application, we can study iterated function systems with place-dependent probabilities. For example, place-dependent Bernoulli convolutions or Blackwell measure for binary channel.

The new results are from a recent paper joint with Balázs Bárány, Boris Solomyak, and Adam Śpiewak.