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Renormalization in Dimension Two

Loosely speaking, a dynamical system is renormalizable if it exhibits self-similarity at a smaller scale. Understanding this phenomenon often yields deep results about the combinatorial, topological and geometric nature of the dynamics. However, the existing techniques are largely limited to one-dimensional cases (e.g. maps on an interval, a circle, a domain in $\mathbb{C}^1$, etc).

In this mini-course, we will extend the renormalization approach to a higher-dimensional setting by combining it with the theory of non-uniformly partially hyperbolic systems. More specifically, we will generalize the renormalization theory of unimodal interval maps to a certain class of diffeomorphisms in dimension two. A key step will be to identify the higher-dimensional analog of a "critical point". Our main result will be two-dimensional a priori bounds, which is a certain uniform control on the geometry of the dynamics at arbitrarily small scales.

Below is a preliminary outline of the mini-course.

i) The proof of *a priori* bounds for unimodal interval maps in dimension one.
ii) Quantitaive Pesin theory, and the definition of a critical orbit and unimodality in dimension two.
iii) Linear ordering on the renormalization limit set for 2D unimodal diffeomorphisms.
iv) Regularity of the first return map, and the proof of *a priori* bounds for 2D unimodal diffeomorphisms.

This is based on a joint work with S. Crovisier, M . Lyubich and E. Pujals.