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ANDRZEJ ALEXIEWICZ (1917–1995). A BIOGRAPHY*

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Andrzej Alexiewicz was born on February 11, 1917 in Lwów (now Lviv), a city of great importance for Polish science and culture. Both his parents were well-known doctors and they could provide him with carefree childhood and very good conditions for full development. The Lwów of the twenties and thirties offered various opportunities. At that time, it was a leading cultural and scientific centre, with many prominent scientists and artists, people of broad horizons and interests. Among them, there were people as extraordinary as Leon Chwistek (a philosopher, logician, mathematician, art theoretician and painter). There were many outstanding teachers and people who could serve as role models. There were also superb educational institutions. Jan Kazimierz University (the third oldest university in Poland) and the excellent Lwów Polytechnic had international recognition.

After completing his secondary education at Lwów Classical Gymnasium VI, Alexiewicz started his studies at the Faculty of Mathematics and Science of Jan Kazimierz University. In the first year, he took physics, but later devoted himself completely to "the queen of the sciences". He studied mathematics at Jan Kazimierz University at the time when it was the world centre of functional analysis, successfully developed by some of the most prominent Polish mathematicians, universally recognized by the world scientific elite. Alexiewicz's teachers included celebrities like Auerbach, Banach, Kaczmarz, Mazur, Orlicz, Schauder, Steinhaus and Żyliński.

Amazingly, the dramatic time of the second world war, the Soviet and German occupation, neither impeded or inhibited Alexiewicz's scientific development. When the war started, he was a Lwów University student and he was finishing his master's thesis, under Eustachy Żyliński's supervision, who had also supervised Władysław Orlicz's doctoral dissertation. Orlicz was to become Alexiewicz's master and collaborator (together they prepared twelve papers from 1945 to 1969).

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In 1941, Alexiewicz became an assistant lecturer at State Lviv University. After a few months, when the Nazis took over Lwów, he had to abandon his duties. Like many other scientists, he started working at the famous Rudolf Weigl Institute. Apart from this official occupation, he was involved in underground teaching and did intensive scientific research. His skills, diligence and the kindness of Lwów scientists helped him prepare a doctoral dissertation, which seemed impossible at that time. Completing the thesis was a great success of Alexiewicz and his supervisor, Władysław Orlicz. Alexiewicz defended his thesis, titled *On sequences of operators*, in August 1944, before a committee consisting of Zierkhoffer, Orlicz and Nikliborc.

In the course of 1944 and 1945, Alexiewicz came to realize that Lwów would not be a part of after-war Poland. He decided to move to Poznań, together with his wife and children. Władysław Orlicz also arrived in Poznań to hold a chair at Poznań University, which he had received in 1937. They faced enormous organizational and teaching workload in Poznań. The infrastructure of Poznań University was almost entirely destroyed and the teaching staff was scarce, which made the work very difficult. Considering the shortage of teachers and facilities, it was hard to educate hundreds of students, not only at the Faculty of Mathematics and Science.

In July 1945, Alexiewicz became a doctor of science—the Board of the Faculty of Mathematics and Science awarded him a doctorate on the basis of the dissertation he had completed and defended in Lwów. Despite being burdened with teaching responsibilities and organizing academic life, Alexiewicz did not neglect his scientific work. In 1948, he habilitated, upon presenting the thesis *On the Denjoy integral of abstract functions*, and he was conferred the title of "docent". Moreover, he found time to help Władysław Orlicz organize and complete some material left by Stefan Banach, which eventually came out as the book *Introduction to the theory of functions of a real variable*, published in 1951, as volume 17 of the series *Monografie Matematyczne*.

Six years after being habilitated, Alexiewicz became associate professor, and since 1962 he was full professor.

Alexiewicz held responsible positions at Poznań University. From 1951 to 1956, he was an associate dean, and then the dean of the Faculty of Mathematics, Physics and Chemistry. He held the office of vice-Rector at Adam Mickiewicz University for one term (1956–1959), chosen by members of the academic community, and not by administrative appointment. Since 1961, he headed the Second Chair of Mathematics in the Faculty of Mathematics, Physics and Chemistry. He put a lot of energy and effort into organizing extramural studies and affiliated institutions, and he was in charge of them for many years. Finally, in 1969, he became the Head of the Institute of Mathematics. He held this post until he retired in 1987.

Alexiewicz took every opportunity to broaden the scope of his research. He worked for the Institute of Mathematics of the Polish Academy of Sciences, and after receiving a scholarship from the Rockefeller Foundation, he spent the end of 1959 and the beginning of 1960 in the United States.

Alexiewicz's academic output includes 56 papers published in various journals, and an extensive book, *Functional Analysis*, published in Polish as volume 49 of *Monografie*

Matematyczne in 1969. The book presents the basics of Banach space theory, non-locally convex and non-metrizable topological vector spaces, and numerous examples of how methods of functional analysis can be applied. Alexiewicz is also the author of a course book, *Differential Geometry* (in Polish), published in 1966 and 1970, which became a great help to students of mathematics.

Another successful sphere of Alexiewicz's activity was teaching the research personnel. He supervised 18 doctors (more than half of them attained habilitation, and some became professors); he also supervised a huge number of master's theses on various topics.

Many of Alexiewicz's students (especially from the forties and fifties) have pointed out his enthusiasm and the dynamism of his lectures, his tendency to provoke discussion with talented and inquisitive students, and his skill at sharing knowledge in a very attractive way. Some of his outstanding students have remembered in particular his excellent monographic lecture on the Lebesgue integral, which he was delivering for two terms in the academic year 1949/50. He was able to offer a kind and selfless help to everybody who was interested in mathematics; he referred them to relevant literature and faced them with various problems worth solving or analyzing.

Andrzej Alexiewicz continuously supported the Polish Mathematical Society. He was the president of the Poznań Branch of the Society from 1981 to 1983. Furthermore, he was involved in publishing, as the editor of the journal *Functiones et Approximatio*, published by the Institute of Mathematics of Adam Mickiewicz University.

Alexiewicz's personality and character did not conform to the stereotype of a scientist. He had wide interests, and although mathematics was foremost in his mind and heart, he was always sensitive to life charms and beauty which he was able to appreciate and sometimes surrender to. He reacted strongly to various events taking place around him. He often commented publicly on some facts and people whose conduct was at odds with his moral system, and was explicit in his words. He was keen on classical music, and he had a passion for painting. He created realistic and abstract paintings for many years, and he was highly regarded by experts, especially for his use of colouring.

Like all people from Lwów, he preserved warm feelings for his home town. As soon as it was possible, he co-founded the Lwów and South-Eastern Borderlands Lovers' Society, and he was the president of the society for many years.

Andrzej Alexiewicz died on July 11, 1995. He was a colourful, extraordinary and unique person. Therefore he and his achievements will continue to be remembered by his friends, students, collaborators and followers.

Throughout his career, Alexiewicz's main scientific interests focused on broadly understood analysis. This was determined by his contact with Lwów School of Mathematics. It was also influenced by his long-lasting contact and collaboration with Władysław Orlicz.

Alexiewicz's scientific work concerns the following subjects:

- scalar and vector measurable functions,
- sequences of linear operators,
- Denjoy integral,
- differentiation of vector functions,
- differential equations and equations with vector functions,

- two-norm spaces and two-norm algebras and their applications in summability theory,
- analytic functions,
- applications of functional analysis to classical problems of mathematical analysis.

This work is presented very briefly below. The reason for the brevity is the fact that the Faculty of Mathematics and Computer Science of Adam Mickiewicz University is soon going to publish a thorough study *Serta mathematica Andreae Alexiewicz*[1], prepared by Alexiewicz's eleven colleagues, students and collaborators. Each of them presents at length Alexiewicz's findings and ideas, frequently together with some results which preceded Alexiewicz's publications or appeared at the same time, or which were inspired by Alexiewicz's work. The authors of the present article would like to thank the editors, Lech Drewnowski and Zbigniew Palka, and the authors, Jerzy Kąkol, Michał Kisielewicz, Ireneusz Kubiaczyk, Mieczysław Mastyło, Julian Musielak, Marek Nawrocki, Paulina Pych-Taberska, Zbigniew Semadeni and Andrzej Sołtysiak, for making the texts available; we found them very useful.

Scalar and vector measurable functions. Andrzej Alexiewicz paid much attention to the problem of measurability of functions. His first 1945 paper [1], joint with Orlicz, contains an elementary proof of M. Fréchet's theorem that every measurable solution to the Cauchy equation $f(x+y) = f(x) + f(y)$, where $x, y \in \mathbb{R}$, is continuous. His next paper [2] concerns the Hausdorff class of vector functions and refers to the Baire classification of functions. Paper [5] (joint with Orlicz) also concerns the Baire functions. In particular, it is shown that every function f from a metric space to a Banach space X, with a separable range, which is weakly continuous with respect to some fundamental set of continuous functionals over X, belongs to the first Baire class (this result can be extended to the Baire classes of an arbitrary rank). Paper [15] concerns the problem of integrability (in the Riemann sense) of functions from an interval to a Banach space. In this paper we can find an example of a Riemann integrable function $f : [0,1] \to C[0,1]$ that is not weakly continuous at any point of this interval. Alexiewicz also considered the problem of continuity of a separable-valued function $f : [a,b] \to X$ (X is a Banach space) of bounded variation. In [16] it is shown that (for a separable Banach space) f is weakly continuous outside a countable set. This is a generalization of a 1950 result G. Sirvint. In [28] Alexiewicz also extended B. J. Pettis's criterion of Bochner measurability of functions.

Sequences of linear operators. In a series of papers ([3], [4], [8], [10], [12], [13], [20], [29]) Alexiewicz considered problems relating to: operators from the space of bounded measurable functions (scalar-valued and vector-valued) to F-normed spaces; general theories of sequences of operators, where one can get analogues of classical results on sequences of operators (like continuity of the limit operator, condensation of singularities and others); the structure of sequences of operators with values in the space of measurable

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functions. This study was motivated and inspired by numerous earlier results obtained by Haar, Hahn, Lebesgue, and later, by the Lvov School and Fichtenholz.

Among theorems contained in the above cycle of papers we should mention theorems on integral representation of linear operators from $L_\infty(\mu)$ to F-normed spaces Y that are norm continuous or continuous with respect to other convergences. Those theorems establish a correspondence between those types of continuous operators and the properties of the representing measures. In this way Alexiewicz joined in a research area, very popular mainly in the late thirties and the fourties, concerning the integral representation of operators with respect to vector measures—this integral approach soon became standard.

Other papers ([3], [8]) contain a series of results stating that the pointwise limit of a sequence of linear operators that are continuous with respect to some convergences in $L_\infty(\mu)$ and Y is also continuous with respect to these convergences. We can also find there theorems on condensation of singularities of operators continuous with respect to two-norm convergence. Some of those results concerning operators from $L_\infty(\mu)$ to Y were generalized (using other methods) to the case of Bochner spaces $L_\infty(\mu, X)$, where X is a Banach space.

The analysis of classical theorems on sequences of operators made by Alexiewicz resulted in studies of a general class of continuous operators between complete metric spaces (where the range space is at least an F-space), namely, classes K such that for any sequence (T_n) in K convergence in a residual set implies equicontinuity, or convergence in a set of the second Baire category implies equicontinuity, or convergence in some ball implies convergence on the whole set.

Paper [8] concerns the properties of the sets of points of boundedness of sequences of operators, and some applications. The notion of a Saks pseudogroup is introduced, embracing the class of F-spaces and finite measure spaces. An application of general theorems to the class of additive operators between Saks pseudogroups and F-spaces allowed Alexiewicz to prove Vitali–Hahn–Saks type theorems for F-space-valued measures, and theorems on condensation of singularities for those measures. It also resulted in interesting applications to unconditionally convergent series (Orlicz–Pettis type theorems) and Mazur–Orlicz polynomial operators.

Paper [10] is devoted to analysis of conditions on spaces X and Y equipped with sequential convergences guaranteeing, for sequences of operators $T : X \to Y$ (continuous with respect to these convergences), the continuity of the limit operator, condensation of singularities, and "resonance" (convergence on a dense set implies convergence everywhere). In the proofs the "gliding hump" method was used and this study resulted in distinguishing 10 properties that are essential to this sphere of problems. Alexiewicz verified which of these properties hold in concrete spaces with convergence (in particular, F-spaces with F-norm convergence and two-norm convergence, vector lattices with order convergence, Banach spaces with weak convergence, spaces of scalar-valued and vector-valued measures).

Three papers [13], [20], [21], testifying to real mathematical talent of Alexiewicz, are devoted to a generalization of Saks's results on sequences of operators from some space

X to $L^0(\mu)$ (the space of scalar measurable functions with convergence in measure). Alexiewicz noted that a sequence of operators $T_n : X \to L^0(\mu)$ can be treated as an operator $T : X \to L^0(\mu, s)$ given by the equality $T(x) = (T_n(x))$, where $L^0(\mu, s)$ denotes the set of μ-equivalence classes of Bochner measurable functions with values in the space s of all sequences. Alexiewicz considers continuous linear operators $T : X \to L^0(\mu, X)$, where X, Y are F-spaces, and he obtains the following theorem [20, Theorem 1]:

Let $T : X \to L^0(\mu, X)$ be a linear operator. Then for every Borel linear subspace R of Y there exist a set A belonging to the s-algebra and a residual set Z in X such that:

(a) *for every $x \in X$, $T(x, t) \in R$ almost everywhere in A,*
(b) *for every $x \in Z$, $T(x, t) \notin R$ almost everywhere in A'.*

Considerations of this type were extended by Alexiewicz to the setting where X is a separable F-space, and $L^0(\mu, X)$ is replaced by an arbitrary F-space of measurable functions ([21]).

Denjoy integral. Denjoy integral was the subject of Alexiewicz's habilitation dissertation, entitled *On the Denjoy integral for abstract functions.* He was conferred the degree of *habilitated doctor* at Poznań University in 1948. In two papers concerning this subject he found the general form of continuous linear functionals over the spaces of real functions (defined on an interval) which are integrable in the sense of Denjoy. Every such functional is of the form $f(x) = (D)\int_a^b x(t)h(t)dt$, where h is a right continuous function of finite variation such that $h(0) = 0$. In [11] he dealt with the Denjoy–Bochner integral of Banach space-valued functions. He established that if a function is integrable in the Denjoy–Bochner sense and its Denjoy–Bochner integral is absolutely continuous, then the function is integrable in the sense of Lebesgue–Bochner. Moreover, the integrability in the sense of Denjoy-Bochner on each measurable subset of the interval means integrability in the sense of Lebesgue–Pettis and the identity of these integrals on every measurable set. Paper [11] contains an example of a function which is integrable both in the sense of Denjoy–Bochner and Lebesgue–Pettis but it is not integrable in the sense of Lebesgue–Bochner.

Differentiability of vector functions. In [9] Alexiewicz studied connections between different kinds of differentiability of functions $f : E \to X$, where E denotes an open subset of the set of real numbers, and X is a Banach space. In particular, he showed the following implication: if f is weakly differentiable in E with derivative $g : E \to X$, then f is strongly differentiable with derivative g almost everywhere in E. Alexiewicz also generalized a result of T. Ważewski (concerning the mean value theorem) to the case of weakly differentiable functions. Paper [18], joint with Orlicz, presents characterizations of sets on which vector-valued functions are differentiable. Using a Baire category argument it is proved that if a function f from an open subset of a separable Banach space to a Banach space is continuous and its quasi-differential exists at every point of its domain, then on some residual subset of the domain there also exists a continuous Gateaux differential of this function.

Differential equations and equations with vector functions. Alexiewicz's interests also embraced partial differential equations and equations with vector functions. This subject is studied in [24], [26], [38], [39]. In the first of these, one can find a proof of the existence of a solution of the equation $y = \phi(t, y)$ with the initial condition $y(\tau) = \eta$, where ϕ is continuous with respect to the second variable, measurable with respect to the first variable and has an integrable majorant $s(t) \geq |\phi(t, u)|$.

In the second paper he considered the problem of the existence of a solution of a differential equation $\frac{\partial^2 z}{\partial x \partial y} = f(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})$ with boundary conditions, and of continuous dependence of a solution on boundary conditions. It was shown that the set of solutions is of the first Baire category. A continuation of [26] is [38], where a more general setting with a Banach space-valued function f is considered.

The last paper in this series is devoted to the equation $x = \phi(x, z)$, where $\phi : K \times Z \to W$ is a continuous function, Z denotes a metric space, and W is a compact subset of a closed convex set K in a Banach space. The main problem considered in [39] is continuous dependence of the solution on a parameter.

Two-norm spaces, two-norm algebras and their application to the theory of summability. For many years Alexiewicz was interested in two-norm spaces, that is, linear spaces X with two norms $\|\cdot\|, \|\cdot\|^*$ (or a norm $\|\cdot\|$ and a locally convex topology τ^*), where the first norm is stronger than the second (the $\|\cdot\|$-norm topology is stronger than τ^*). Basic concepts relating to these spaces include γ-convergence ($x_n \xrightarrow{\gamma} x$ means that x_n is convergent to x with respect to $\|\cdot\|^*$ and (x_n) is bounded with respect to $\|\cdot\|$) and γ-normality (balls in $\|\cdot\|$ are closed in $\|\cdot\|^*$-topology).

The study of two-norm spaces in [1], [23], [31], [32], [34], [36], [40] is well motivated: this class includes many important and interesting spaces. Among the important results obtained by Alexiewicz we mention theorems on connections between the spaces X'_*, X'_γ, X' of linear functionals on X that are $\|\cdot\|^*, \gamma, \|\cdot\|$-continuous respectively. If a two-norm space X is γ-normal, then X'_γ is equal to the closure of X'_* in X'. This fact enabled a uniform description of X'_γ for many concrete spaces X.

Alexiewicz also considered the problem of extension of γ-continuous linear functionals from γ-closed subspaces (which is not always possible; Alexiewicz gives sufficient conditions on a two-norm space, guaranteeing the existence of such extensions). He also studied the structure of two-norm spaces: their γ-completeness, γ-reflexivity, γ-separability and γ-compactness.

The theory of two-norm spaces is strictly connected with the theory of Saks spaces (developed by Orlicz) and has had numerous applications—in particular, in two-norm algebras introduced by Alexiewicz in the seventies, that is, algebras X with a two-norm structure such that multiplication is continuous with respect to the γ-convergence. In [47] Alexiewicz formulated sufficient conditions for this continuity under the assumption of continuity with respect to each variable separately. Papers [47], [49] and [50] are also devoted to the study of the representation of continuous linear multiplicative functionals. An essential role in the theory of two-norm spaces is played by the so called Wiweger topology τ_γ, which is locally convex and topologizes γ-convergence, that is, for a sequence (x_n) we have $x_n \xrightarrow{\gamma} x$ if and only if $x_n \to x$ for τ_γ. In [51] Alexiewicz gave a characterization of continuity of multiplication in two-norm algebras with the Wiweger topology, and in [48] he obtained sufficient and necessary conditions for continuity of the inverse in two-norm algebras.

Two-norm spaces were the starting point for his research in the theory of inductive limits of locally convex topological algebras. An important achievement in this area is a characterization of multiplicativity of the inductive limit topology of a generalized inductive system (see [51], [54], [55]).

Important applications of the theory of two-norm spaces include generalized Mazur–Orlicz consistency theorems for matrix methods of summability. Alexiewicz and Orlicz considered in [33] the consistency of linear methods of summability of sequences in a Banach space X, replacing numerical matrices by matrices of continuous linear operators from X to a Banach space Y. This research was continued in [35] and [55]. In [25] Alexiewicz also obtained consistency results for methods of summability of double sequences.

Analytic functions. Alexiewicz studied both real and complex analytic functions ([14], [19], [22], [45], [46], [52]). His results relate to: connection between analyticity and weak analyticity of functions with values in a Banach space X (with respect to a fundamental system of continuous linear functionals over X); sets of uniqueness for functions $f : T \to X$; an analogue of Vitali's theorem for holomorphic vector-valued functions; a general construction of the so called simultaneous automorphisms; integral representations of γ-continuous functionals on two-norm spaces connected with the Hardy spaces H^1 and H^∞.

Applications of functional analysis to classical problems of mathematical analysis. The methods of functional analysis can be used to obtain simple and elegant solutions and generalizations of problems of mathematical analysis. This is evidenced in [6] and [27], where it was shown how successfully the theory of Banach spaces acts in the field of number series. Paper [27] shows how elementary means of the theory of Banach spaces connected with a very good idea and a proper interpretation of concepts can essentially enrich the classical theory.

The scientific output of Andrzej Alexiewicz indicates his immense creativity and intellectual power. It reveals his ability to understand the core of a problem, grasp the interrelations and dependencies between different notions, and solve natural, well-motivated problems in a clear and elegant way. It is a great pity that his achievements will not be enriched any more.

List of publications of Andrzej Alexiewicz

[1] (with W. Orlicz) *Remarque sur l'équation fonctionnelle* $f(x+y) = f(x) + f(y)$, Fund. Math. 33 (1954), 314–315.

[2] *On Hausdorff classes*, Fund. Math. 34 (1946), 61–65.

[3] *Linear operations among bounded measurable functions I*, Ann. Soc. Polon. Math. 19 (1947), 140–160.

[4] *Linear operations among bounded measurable functions II*, Ann. Soc. Polon. Math. 19 (1947), 161–164.

[5] (with W. Orlicz) *Sur la continuité et la classification de Baire des fonctions abstraites*, Fund. Math. 35 (1948), 105–126.

[6] *On multiplication of infinite series*, Studia Math. 10 (1948), 104–112.

[7] *Linear functionals on Denjoy-integrable functions*, Colloq. Math. 1 (1948), 289–293.

[8] *On sequences of operations (I)*, Studia Math. 11 (1949), 1–30.

[9] *On differentiation of vector-valued functions*, Studia Math. 11 (1949), 185–196.

[10] *On sequences of operations (II)*, Studia Math. 11 (1949), 200–236.

[11] *On Denjoy integrals of abstract functions*, Soc. Sci. Lett. Varsovie C.R. Cl. III Sci. Phys. 41 (1948), 97–129. (1950)

[12] *On sequences of operations (III)*, Studia Math. 12 (1951), 84–92.

[13] *On sequences of operations (IV)*, Studia Math. 12 (1951), 93–101.

[14] (with W. Orlicz) *On analytic vector-valued functions of real variable*, Studia Math. 12 (1951), 108–111.

[15] (with W. Orlicz) *Remarks on Riemann-integration of vector-valued functions*, Studia Math. 12 (1951), 125–132.

[16] *Continuity of vector-valued functions of bounded variation*, Studia Math. 12 (1951), 133–142.

[17] *On a theorem of Ważewski*, Ann. Soc. Polon. Math. 24 (1951), 129–131.

[18] (with W. Orlicz) *On the differentials in Banach spaces*, Ann. Soc. Polon. Math. 25 (1952), 95–99.

[19] *On the localization of values of vector-valued functions*, Ann. Soc. Polon. Math. 25 (1952), 288–297.

[20] *On some theorems of S. Saks*, Studia Math. 13 (1953), 18–29.

[21] *A theorem on the structure of linear operations*, Studia Math. 14 (1953), 1–12.

[22] (with W. Orlicz) *Analytic operations in real Banach spaces*, Studia Math. 14 (1953), 57–78.

[23] *On the two-norm convergence*, Studia Math. 14 (1953), 49–56.

[24] (with W. Orlicz) *On a theorem of C. Carathéodory*, Ann. Polon. Math. 1 (1955), 414–417.

[25] (with W. Orlicz) *On summability of double sequences (I)*, Ann. Polon. Math. 2 (1955), 170–181.

[26] (with W. Orlicz) *Some remarks on the existence and uniqueness of solutions of the hyperbolic equation* $\frac{\partial^2 z}{\partial x \partial y} = f(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})$, Studia Math. 15 (1956), 201–215.

[27] *On Cauchy's condensation theorem*, Studia Math. 16 (1957), 80–85.

[28] *On certain "weak" properties of vector-valued functions*, Studia Math. 17 (1958), 65–68.

[29] *Addition to the paper "On some theorems of S. Saks"*, Studia Math. 17 (1958), 69.

[30] *On unconditional convergence in spaces conjugate to a Banach space*, Bull. Soc. Amis Sci. Lettres Poznań, Sér. B 14 (1958), 5–8.

[31] (with Z. Semadeni) *Linear functionals on two norm spaces*, Studia Math. 17 (1958), 121–140.

[32] (with Z. Semadeni) *A generalization of two norm spaces. Linear functionals*, Bull. Acad. Polon. Sci. 6 (1958), 135–139.

[33] (with W. Orlicz) *Consistency theorems for Banach space analogues of Toeplitzian methods of summability*, Studia Math. 18 (1959), 199–210.

[34] (with Z. Semadeni) *The two-norm spaces and their conjugate spaces*, Studia Math. 18 (1959), 275–293.

[35] (with W. Orlicz) *On summability of double sequences (II)*, Ann. Polon. Math. 6 (1959), 171–180.

[36] (with Z. Semadeni) *Some properties ot two-norm spaces and a characterization of reflexivity of Banach spaces*, Studia Math. 19 (1960), 115–132.

[37] (with W. Orlicz) *Inequalities for functionals in locally convex linear spaces*, Zeszyty Nauk. Uniw. A. Mickiewicza 25 (1960), Mat. Fiz. Chem., Z. 2, 9–13.

[38] (with J. Kopeć) *The equation $y' = f(t, y)$ for vector-valued functions. Dependence of the solutions on initial data*, Zeszyty Nauk. Uniw. A. Mickiewicza 25 (1960), Mat. Fiz. Chem., Z. 2, 15–20.

[39] *Dependence of fixed points on a parameter*, Zeszyty Nauk. Uniw. A. Mickiewicza 25 (1960), Mat. Fiz. Chem., Z. 2, 21–26.

[40] *The two-norm spaces*, Studia Math., Ser. Spec. Z. 1 (1963), 17–20.

[41] (with M. G. Arsove) *Simultaneous automorphisms in spaces of analytic functions*, Canad. J. Math. 15 (1963), 495–502.

[42] *Cybernetyka–nowa nauka*, Uniw. im. A. Mickiewicza w Poznaniu, Seria Wykłady Inauguracyjne Nr 7 (1968).

[43] *Analiza Funkcjonalna*, PWN, Warszawa, 1969.

[44] *A topology for two-norm spaces*, Funct. Approx. Comment. Math. 1 (1974), 1–8.

[45] *The Hardy space H^1 as a two-norm space*, Funct. Approx. Comment. Math. 1 (1974), 9–16.

[46] (with A. Golusda) *The two-norm space of bounded holomorphic functions*, Funct. Approx. Comment. Math. 1 (1974), 17–25.

[47] *On some two-norms algebras*, Funct. Approx. Comment. Math. 2 (1976), 3–34.

[48] *Two-norm algebras*, in: Proc. Fourth Prague Topological Symp. (1976), Part B, 21–24.

[49] *On some two-norms algebras*, Funct. Approx. Comment. Math. 3 (1976), 3–10.

[50] *Some two-norms algebras with γ-continuous inverse*, Funct. Approx. Comment. Math. 3 (1976), 11–21.

[51] *The Wiweger topology in two-norm algebras*, Comment. Math. T. Spec. in honorem L. Orlicz I (1978), 19–31.

[52] *An interpolation theorem for holomorphic functions*, in: Topology, Vol. II (Proc. Fourth Colloq., Budapest, 1978), Coll. Math. J. Bolyai 23 (1980), 35–39.

[53] *Linear algebras with the topology of Roelcke*, Funct. Approx. Comment. Math. 7 (1979), 135–138.

[54] (with J. Kąkol) *Garling algebras*, Funct. Approx. Comment. Math. 8 (1980), 111–118.

[55] (with M. Szmuksta-Zawadzka) *Two-norm spaces and summability*, Note di Mat. 9 (1989), 267–277.

[56] (with M. Szmuksta-Zawadzka) *Waelbroeck sequences*, Comment. Math. Prace Mat. 31 (1991), 1–8.