

## Interaction of Waves for Dispersion-less Nonlinear First Order Systems of PDE,s

Zbigniew Peradzyński

In this lecture we will consider first order homogeneous nonlinear systems of partial differential equations

$$A_j^{sv}(u)u_{,v}^j = 0, \quad \text{where } u = (u^1(x), u^2(x), \dots, u^l(x)), \quad x \in R^n$$

All consideration will have local character. If the system is non-elliptic then it has special solutions representing propagating nonlinear waves – Riemann waves (simple waves). The first solution representing interaction of two such a waves (two sound waves in gasdynamics) was considered by G.B. Riemann. This sort of interaction can be viewed as elastic since no new waves are appearing during the interaction. We define “elastic interaction” of waves and show that it is related to the existence of special system of coordinates in the space of dependent variables - Riemann invariants. In general we can have a certain number, say  $k$  of interacting elastically waves. Such solutions are known as  $k$ -tuple waves ( or  $k$ - waves). There are cases with  $k = \infty$ . However in general the interaction does not have to be elastic. Then as will be shown new waves can be generated. The main tool to study these processes is the Cartan theory of integrability.