Some Bourgain-Brezis type solutions via complex interpolation

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Abstract

In 2002 Bourgain and Brezis proved that given a vector field $v \in \mathcal{S}'(\mathbb{R}^d) \cap \dot{W}^{1,d}(\mathbb{R}^d)$ there exists a vector field $u \in L^\infty(\mathbb{R}^d) \cap \dot{W}^{1,d}(\mathbb{R}^d)$ such that $\operatorname{div} u = \operatorname{div} v$. We prove several results of a similar nature in which we take into consideration the Fourier support of the solutions. For instance, in the case $d \geq 3$ we prove the following: for any vector field $v \in \mathcal{S}'(\mathbb{R}^d) \cap \dot{B}_q^{d/p,p}(\mathbb{R}^d)$ (where $p \in [2,\infty)$ and $q \in (1,2)$), with *supp* $\widehat{v} \subseteq \mathbb{R}^d \backslash (-\infty,0)^d$, there exists a vector field $u \in L^\infty(\mathbb{R}^d) \cap \dot{B}_2^{d/p,p}(\mathbb{R}^d)$, with *supp* $\widehat{v} \subseteq \mathbb{R}^d \backslash (-\infty,0)^d$, such that

$$\operatorname{div} u = \operatorname{div} v,$$

and

$$\|u\|_{L^\infty \cap \dot{B}_2^{d/p,p}} \lesssim \|v\|_{\dot{B}_q^{d/p,p}}.$$

Our arguments rely on a version of the complex interpolation method combined with some ideas of Bourgain and Brezis.

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