Liouville–Arnol'd theorem for contact Hamiltonian systems

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Contact dynamics has gained popularity as a geometric formalism for modelling certain dynamical systems with dissipation. The category of contact manifolds is equivalent to the category of $\mathbb{R}^\times$-principal bundles endowed with 1-homogeneous symplectic forms. Moreover, contact Hamiltonian vector fields are in one-to-one correspondence with symplectic Hamiltonian vector fields of 1-homogeneous functions.

The celebrated Liouville–Arnol'd theorem states that if on a $2n$-dimensional symplectic manifold (M, ω) there are n independent functions f_i, $i \in \{1, \ldots, n\}$ in involution (i.e., their Hamiltonian vector fields commute with each other), then M is foliated by Lagrangian submanifolds which are left invariant by the Hamiltonian flows of f_i, and there are action-angle coordinates such that i) they are canonical coordinates for ω; ii) action coordinates specify the Lagrangian submanifold; iii) angle coordinates have "constant speed" with respect to the Hamiltonian dynamics of each f_i, while action coordinates are constants of the motion. Several attempts (by Boyer, Jovanović, and other authors) have been made to generalise this result and extend the theory of integrable systems to the realm of contact manifolds. However, there assumptions are so restrictive that one cannot consider Hamiltonian functions leading to dissipation of energy. Instead, we have proven a Liouville–Arnol'd theorem for homogeneous functions on homogenous symplectic manifolds, and deduced from it a counterpart for contact manifolds.

References

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