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Diffusion process with small diffusion in Markov media

Consider a stochastic system with diffusion perturbation, which is a solution of the stochastic differential equation

$$du^\varepsilon(t) = C\big(u^\varepsilon(t); x(t/(\delta\varepsilon))\big)\,dt + \delta^{1/2}\sigma\big(u^\varepsilon(t); x(t/(\delta\varepsilon))\big)\,dw(t), \tag{1}$$

where $u^\varepsilon(t)$ — random evolution [1]; $C(u;x)$, $u \in R^d$ — regression function [2]; $w(t)$ — Wiener process; $\sigma(u;x)$ — diffusion; δ — small parameter which characterizes effect of diffusion on random evolution. Uniformly ergodic Markov process $x(t)$, $t \geqslant 0$, defined on the phase space of states $(X, \mathbf{X})$ with stationary distribution $\pi(B)$, $B \in \mathbf{X}$, and generator $Q\varphi(x) = q(x)\int_X P(x,dy)[\varphi(y) - \varphi(x)]$ on real-valued test-functions with norm $\|\varphi(x)\| := \sup_{x\in X}\varphi(x)$ [1]. Here $q(x)$ — intensity of the time spent in the system state x.

Theorem. *For the diffusion process* (1) *the weak convergence* $u^\varepsilon(t) \Rightarrow \widehat{u}(t)$, $\varepsilon \longrightarrow 0$, *holds, where limited evolution* $\widehat{u}(t)$ *is a solution of the stochastic differential equation* $d\widehat{u}(t) = C(\widehat{u}(t))\,dt + \delta^{1/2}\sigma(\widehat{u}(t))\,dw(t)$ *with shift* $C(u) = \int_X \pi(dx)\,C(u;x)$ *and diffusion* $\sigma^2(u) = \int_X \pi(dx)\sigma^2(u;x)$.

References

[1] V. Koroliuk, N. Limnios, *Stochastic Systems in Merging Phase Space*, World Scientific Publishing, 2005.

[2] U. T. Khimka, Y. M. Chabanyuk, *Convergence of stochastic optimization procedure with Markov switching*, (PDMU-2009) International Conference. Abstracts. Schidnizya, Ukraine, April 27–30, 2009. P. 172.