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Large deviations for random evolutions with global balance conditions

Consider large deviations for random evolution in asymptotic small diffusion scheme [1,2] with global balance conditions defined by stochastic differential equation [3,4]

$$du^{\varepsilon,\delta}(t) = C(u^{\varepsilon,\delta}(t); x(t/\varepsilon^3))\, dt + (\varepsilon^{-1} + \delta^{-1})C_0(x(t/\varepsilon^3))\, dt, \tag{1}$$

where the uniformly ergodic Markov process $x(t)$, $t \geqslant 0$, is defined on the phase space of states $(X, \mathbf{X})$ with stationary distribution $\pi(B)$, $B \in \mathbf{X}$, [4,5] and the generator $\mathbf{Q}\varphi(x) = q(x) \int_X P(x, dy)[\varphi(y) - \varphi(x)]$ on real-valued test-functions with the norm $\|\varphi(x)\| := \sup_{x\in X} \varphi(x)$ [4]; $u^{\varepsilon,\delta}(t)$ — random evolution [5]; $C(u; x)$, $u \in R^d$ — regression function [3], $\mathbf{R}_0$ potential by operator $\mathbf{Q}$ [4]. The global balance condition holds $\int_X C_0(x)\, \pi(dx) = 0$.

Lemma 1. *The random evolution* (1) *is defined by the generator*

$$\boldsymbol{L}^{\varepsilon,\delta}(x)\varphi(u, x) = [\varepsilon^{-3}Q + (\varepsilon^{-1} + \delta^{-1})\boldsymbol{C}_0(x) + \boldsymbol{C}(x)]\varphi(u, x),$$

where $\boldsymbol{C}_0(x)\varphi(u) := C_0(x)\varphi'(u)$, $\boldsymbol{C}(x)\varphi(u) := C(u; x)\varphi'(u)$.

Lemma 2. *A solution of the singular perturbation problem* [4, 5] *for the generator* $\boldsymbol{L}^{\varepsilon,\delta}(x)$ *on perturbed test-function* $\varphi^{\varepsilon,\delta}(u, x) = \varphi(u) + (\varepsilon + \delta)\varphi_1(u, x) + \varepsilon^3\varphi_2(u, x)$, *where* $\frac{\varepsilon}{\delta} \to 1, \varepsilon \to 0$, *has the form* $\boldsymbol{L}^{\varepsilon,\delta}(x)\varphi^{\varepsilon,\delta}(u, x) = \boldsymbol{L}^\delta\varphi(u) + \theta^\varepsilon_L(x)\varphi(u)$, *where* $\boldsymbol{L}^\delta = \Pi\boldsymbol{C}(x)\Pi + \frac{1}{2}\delta\sigma^2$, $\sigma^2 = 2\Pi\boldsymbol{C}_0(x)\boldsymbol{R}_0\boldsymbol{C}_0(x)\Pi$, *and a negligible term* $\theta^\varepsilon_L(x)\varphi(u)$ *is defined by the ratio* $sup_{x\in X}\theta^\varepsilon_L(x)\varphi(u) \to 0, \varepsilon \to 0$.

Theorem 1. *The exponential generator* $\boldsymbol{H}^\delta\varphi(u) := \exp(-\frac{\varphi(u)}{\delta})\delta\boldsymbol{L}^\delta \exp(\frac{\varphi(u)}{\delta})$ [1, 2] *the limiting evolution* $du(t) = C(u(t))\, dt + \delta^{1/2}\sigma\, dw(t)$ *with the shift* $C(u) = \int_X \pi(dx)C(u; x)$, *converges to the limiting exponential generator* $\boldsymbol{H}\varphi(u) = \frac{1}{2}\sigma^2[\varphi'(u)]^2 + C(u)\varphi'(u)$, $\delta \to 0$.

Corollary 1. *The theorem solves the large deviations problem* [2] *for random evolution* (1).

References

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