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Compensating operator diffusion process with variable diffusion in semi-Markov space

Consider the diffusion process in semi-Markov space with variable diffusion defined by stochastic differential equation [1,2]

$$du^{\varepsilon}(t) = C(u^{\varepsilon}(t); x(t/\varepsilon))\, dt + \sigma(u^{\varepsilon}(t); x(t/\varepsilon))\, dw(t), \tag{1}$$

where $u^{\varepsilon}(t)$ — random evolution in the form of a diffusion process [2]; $x(t)$, $t \geqslant 0$ — semi-Markov process in the standard phase space of $(X, \mathbf{X})$ with stationary distribution $\pi(B)$, $B \in \mathbf{X}$, [2]; ε — small parameter.

Compensating operator [2] can be defined by the relation

$$\mathbf{L}\varphi(x,t) = q(x)\left[\int_0^{\infty} G_x(ds)\int_X P(x,dy)\varphi(y,t+s) - \varphi(x,t)\right],$$

where $G_x(ds)$ — distribution function [2].

Lemma. *A compensating operator* [2] *by the process* (1) *on test-functions* $\varphi(u,x) \in C^3(R,X)$ *is defined by the formula*

$$\begin{aligned}\mathbf{L}^{\varepsilon}(x) &= \varepsilon^{-1}\mathbf{Q}\varphi(u,x) + \theta_1^{\varepsilon}(x)\mathbf{Q}_0\varphi(u,x)\\ &= \varepsilon^{-1}\mathbf{Q}\varphi(u,x) + \mathbf{C}(x)\varphi(u,x) + \varepsilon\theta_2^{\varepsilon}(x)\varphi(u,x),\end{aligned}$$

where $\mathbf{Q}$ *— generator of embedded Markov process* [2]; $\mathbf{Q}_0 = q(x)\int_X P(x,dy)$, $q(x) = \int_0^{\infty}(1 - G_x(t))\,dt$; *limited operators* $\theta_1^{\varepsilon}(x)$, $\theta_2^{\varepsilon}(x)$; $\mathbf{C}(x)\varphi(u,x) = C(u,x)\varphi'(u,x) + \frac{1}{2}\sigma^2(u,x)\varphi''(u,x)$.

References

[1] U. T. Khimka, Ya. M. Chabanyuk, *A difference stochastic optimization procedure with impulse perturbation*, Cybernetics and Systems Analysis 49 (2013), 768–773.

[2] V. Koroliuk, N. Limnios, *Stochastic Systems in Merging Phase Space*, World Scientific Publishing, Hackensack, NJ 2005.