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On the numerical solution of an ODE using an orthogonal wavelet expansion

We treat here a differential equation of an arbitrary order m

$$\sum_{r=0}^{m} a_{m-r}(x)y^{(m-r)}(x) = f(x) \tag{1}$$

for a standard interval $x \in [a, b]$ which is split, for wavelet rank p, into s subintervals:

$$x_k \in [a_k, b_k] = [a + 2^{-p}\alpha_p(b-a), a + 2^{-p}(\alpha_p(b-a)+1)(b-a)] \quad \underset{0\leqslant k=\alpha_p\leqslant 2^p-1}{\forall}$$

$$x_k = a + 2^{-p}[k(b-a) + (x-a)] = a + 2^{-p}[\alpha_p(b-a) + (x-a)] \quad \underset{0\leqslant k=\alpha_p\leqslant 2^p-1}{\forall}$$

where we expand both the known function $f(x)$ and the unknown function $y(x)$ either by orthogonal polynomials and wavelets orthogonal polynomials

$$f(x_k) = f_k(x) = \sum_{j=0}^{n} f_{k,j}\phi_j(x) = \sum_{j=0}^{n} f_{k,j}\phi_{j,p}(x_k) \quad (m \leqslant n) \quad \underset{0\leqslant k\leqslant 2^p-1}{\forall}$$

$$f_{k,j} = \frac{(f_k, \phi_j)_{L^2[a,b]}}{||\phi_j||^2_{L^2[a,b]}} = \frac{(f, \phi_{j,p})_{L^2[a_k,b_k]}}{||\phi_{j,p}||^2_{L^2[a_k,b_k]}} \quad \underset{0\leqslant j\leqslant n}{\forall} \quad \underset{0\leqslant k\leqslant 2^p-1}{\forall}$$

$$y(x_k) = y_k(x) = \sum_{j=0}^{n} y_{k,j}\phi_j(x) = \sum_{j=0}^{n} y_{k,j}\phi_{j,p}(x_k) \quad (m \leqslant n) \quad \underset{0\leqslant k\leqslant 2^p-1}{\forall}$$

$$y_{k,j} = \frac{(y_k, \phi_j)_{L^2[a,b]}}{||\phi_j||^2_{L^2[a,b]}} = \frac{(y, \phi_{j,p})_{L^2_{[a_k,b_k]}}}{||\phi_{j,p}||^2_{L^2[a_k,b_k]}} \quad \underset{0\leqslant j\leqslant n}{\forall} \quad \underset{0\leqslant k\leqslant 2^p-1}{\forall}.$$

Remark that

$$\phi^{(r)}_{j,p}(x_k) = \phi^{(r)}_j(x)\left(\frac{dx}{dx_k}\right)^r = 2^{pr}\phi^{(r)}_j(x) \quad \underset{0\leqslant j\leqslant n}{\forall} \quad \underset{0\leqslant r\leqslant m}{\forall} \quad \underset{0\leqslant k\leqslant 2^p-1}{\forall}$$

After setting these expansions to the differential equation (1) and using the Galerkin Method or Least Squares Method we reduce the problem to a linear algebraic problem easy to treat numerically. Such expansions lead us to p equation systems of order at least $m+1$, where the RHS are easy to obtain, and having a common coefficients matrix when all coefficients of our differential equation are constant. In general case it is not necessary to impose any conditions to the solution of (1) especially for an stiff one. An example of a stiff ODE will be given.