Olech's Theorem on Global Stability and Models of Economic Growth

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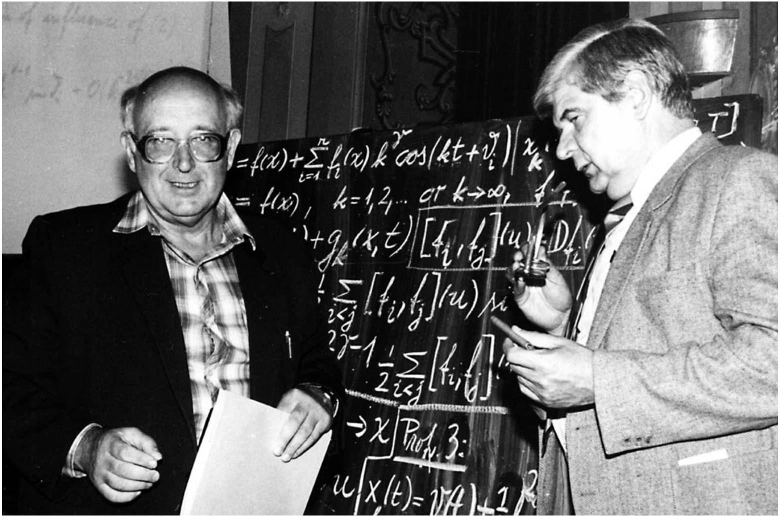

Motto:

Prove simple theorems, those are the most useful.

Olech's early years in Krakow

- In 1949 Czesław Olech started studying mathematics at the Jagiellonian University in Krakow.
- There he met Tadeusz Ważewski who became his mentor.

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- Among his older colleagues there were:
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  Zdzisław Opial,
  Andrzej Plis,
  Jacek Szarski.
- He defended his PhD under the supervision of Ważewski in 1958.

Olech's first visit to US

- 1954, Mathematical Congress in Amsterdam: Tadeusz Ważewski presents his famous topological principle and meets Solomon Lefschetz.
- 1960: Lefschetz invites Olech, recommended by Ważewski, to visit the Research Institute of Advanced Studies (RIAS) created by him in Baltimore.

Olech's first visit to US

- 1954, Mathematical Congress in Amsterdam: Tadeusz Ważewski presents his famous topological principle and meets Solomon Lefschetz.
- 1960: Lefschetz invites Olech, recommended by Ważewski, to visit the Research Institute of Advanced Studies (RIAS) created by him in Baltimore.

During the academic year 1960-1961 in RIAS:

- He is asked to explain the Ważewski principle. (Lefschetz's comment was that the principle was the most important theorem in ODE's obtained, to date, after World War II.)
- He meets Philip Hartman, Lawrence Marcus and Garry Meisters. Later he writes common papers with two of them.

Local and global stability of dynamical systems

- Effective criteria for local stability of autonomous systems of differential equations were first given in the pioneering work of A.M. Lyapunov, published in his thesis in 1892.
- His criterion for local asymptotic stability at an equilibrium: the eigenvalues of the linear part of the right hand side of equations have negative real parts.

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- Many attempts were made later to globalize his local results.

Recall that an equilibrium point x_0 of a dynamical system in IR^n

$$\dot{x} = f(x),$$

is called globally (resp. locally) asymptotically stable if the solution starting from any $x(0) \in IR^n$ (resp. from any $x(0) \in U$, where U is a neighbourhood of x_0) tends to x_0 when time goes to infinity.

Global stability in the plane

Early in the first Olech's stay in RIAS Lawrence Markus came to the Lefschetz seminar presenting latest results on global stability of dynamical systems in IR^n. He also presented an open problem, called later the Markus-Yamabe Conjecture.

Global stability in the plane

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Markus-Yamabe conjecture. Assume that a dynamical system

$$\dot{x} = f(x), \quad x \in IR^n,$$

where the vector field $f(x)$ is of class C^1, satisfies the conditions

(A) $$f(0) = 0,$$

(B) the eigenvalues of the Jacobian matrix $J(x) = \frac{\partial f}{\partial x}(x)$ have negative real parts for all $x \in IR^n$.

Then the system is globally asymptotically stable at the origin.

Olech's theorems in the plane I

After hearing the problem stated by Markus Olech found positive results under some additional assumptions.

Theorem (1)

Suppose that a dynamical system $\dot{x} = f(x)$ in the plane, with $f \in C^1$, satisfies the conditions

(A) $$f(0) = 0,$$

(B) the eigenvalues of the Jacobian matrix $J(x) = \frac{\partial f}{\partial x}(x)$ have negative real parts for all $x \in IR^n$, and

(D) $$\frac{\partial f_1}{\partial x_1}\frac{\partial f_2}{\partial x_2} \neq 0 \ \text{ on } \ IR^2 \quad \text{or} \quad \frac{\partial f_1}{\partial x_2}\frac{\partial f_2}{\partial x_1} \neq 0 \ \text{ on } \ IR^2.$$

Then it is globally asymptotically stable at the origin.

Clearly, condition (B) can be replaced by two conditions

$$\operatorname{tr} J(x) < 0 \quad \forall\, x \in IR^2, \qquad (i)$$

$$\det J(x) > 0 \quad \forall\, x \in IR^2 \qquad (ii)$$

on the Jacobian matrix

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix}$$

Conditions (i), (ii) and (D) are easy to verify.

Theorem 1 with condition (B) replaced by (i) and (ii) is now known in mathematical economy as Olech's Theorem.

Another version proved in the same paper:

Theorem (2)

Suppose that a dynamical system $\dot{x} = f(x)$ *in the plane satisfies the conditions*

(A) $$f(0) = 0,$$

(B) *the eigenvalues of the Jacobian matrix* $J(x) = \frac{\partial f}{\partial x}(x)$ *have negative real parts for all* $x \in IR^n$*,*

(C) $$|f(x)| > \rho \ \text{ for } \ |x| > r$$

for some constants $\rho > 0$, $r > 0$.

Then it is globally asymptotically stable at the origin.

Olech's theorems in the plane III

Still another version of the result, published in the same paper in 1963, was the following generalization of a theorem of Krasovski.

Theorem (3)

Suppose the system $\dot{x} = f(x)$ in the plane satisfies:
(A) it has one singular point and it is a point of attraction,

(B) $$\frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} \leq 0 \quad \text{on} \quad IR^2,$$

(C) $$|f(x)| > \rho \quad \text{for} \quad |x| > r$$

for some constants $\rho > 0$, $r > 0$.

Then it is globally asymptotically stable at the singular point.

The proofs of the above theorems are tricky but elementary.

Solution of Markus-Yamabe Conjecture (MYC) in the plane

Olech's theorems did not solve MYC.
Even if the first two assumptions were equivalent or weaker to those in MYC, the third was an additional one.

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In his 1963 paper Olech considered also the following problem.

Let $n = 2$ and the Jacobian matrix $J_f(x)$ of the vector field f have negative real parts. Is then the map $f : IR^2 \to IR^2$ globally one-to-one?

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He proved, using his Theorem 1, that a positive answer to this problem implies a positive answer to the MYC for $n = 2$.

- In 1988 Meisters and Olech proved MYC in dimension 2 for polynomial vector fields, using Olech's Theorem 2.

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- In 1988 Meisters and Olech proved MYC in dimension 2 for polynomial vector fields, using Olech's Theorem 2.
- The general case with $n = 2$ was solved affirmatively in 1993! Three independent proofs were published by A.A. Glutsuk (1994), R. Fessler (1995), and C. Gutierrez (1995).

Negative solution to Markus-Yamabe Conjecture in IR^n

In dimensions higher then 2 MYC turned out to be false:

- In dimensions ≥ 4 analytic counterexamples were given by Bernat and Llibre in 1996.

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- In dimensions ≥ 4 analytic counterexamples were given by Bernat and Llibre in 1996.
- However, Fournier and Martelli in an unpublished manuscript observed that a positive solution to MYC for polynomial vector fields and all n would imply the Jacobian Conjecture.
- Thus, in order to prove the Jacobian Conjecture it was enough to prove MYC for polynomial vector fields!
- Unfortunately, in 1997 A. Cima, A. van den Essen, A. Gassul, E.-M.G.M. Hubbers, and F. Mañosas found polynomial counterexamples to MYC in any dimension ≥ 3.

PART II
Olech's theorem in mathematical economics

With a short search in internet I have found several textbooks in Mathematical Economics stating explicitly Olech's Theorem 1. There are also several research papers with Olech's theorem in the title.

The reasons that it is so useful seem to be:

- The dynamical models of economic growth are usually reduced to two nonlinear ODEs of first order (higher dimensional systems are difficult to analyse).
- Global stability is a highly desirable phenomenon in economic growth.
- The criteria of Olech's theorem seem the simplest to verify among currently available (finding a global Lyapunov function is often a difficult task).

The Mankiw-Romer-Weil model of economic growth

In 1992 N.E. Mankiw, D. Romer and D.N. Weil proposed the following model of economic growth

$$\dot{k} = s_k k^\alpha h^\beta - c\, k,$$

$$\dot{h} = s_h k^\alpha h^\beta - c\, h,$$

where k - physical capital per worker, h - human capital per worker,
$s_k,\ s_h$ - represent ratios of savings invested for increasing physical and human capital,
$c > 0$ - represents, roughly, depreciation rate of physical capital.
It is assumed:

$$\alpha,\ \beta > 0, \quad \alpha + \beta < 1, \qquad s_k,\ s_h > 0, \quad s_k + s_h < 1.$$

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It is assumed:

$$\alpha,\ \beta > 0,\quad \alpha + \beta < 1, \qquad s_k,\ s_h > 0,\quad s_k + s_h < 1.$$

The production (output) per worker is given by

$$y = Ak^\alpha h^\beta.$$

Global stability in Mankiw-Romer-Weil model I

In logarithmic coordinates:

$$K = \ln k,\ \ H = \ln h, \quad S_K = \ln s_k,\ \ S_H = \ln s_h,\ \ C = \ln c$$

the M-R-W model becomes

$$\dot{H} = \exp(S_K + (\alpha - 1)K + \beta H) - \exp C,$$
$$\dot{K} = \exp(S_H + \alpha K + (\beta - 1)H) - \exp C.$$

Here $(K, H) \in IR^2$. Its steady state is determined by linear equations

$$(\alpha - 1)K + \beta H = C - S_K,$$
$$\alpha K + (\beta - 1)H = C - S_H.$$

The determinant of the corresponding matrix is nonzero:

$$\det A = 1 - \alpha - \beta \neq 0. \qquad \text{Denoting} \quad d = (1 - \alpha - \beta)^{-1}$$

we see that the equations have a unique solution (steady state):

$$K^* = d(-C + (1-\beta)S_K + \beta S_H), \quad H^* = d(-C + \alpha S_K + (1-\alpha)S_H).$$

Apply Olech's theorem for the system

$$\dot{H} = g_K - \exp C, \quad \text{where} \quad g_K = \exp(S_K + (\alpha - 1)K + \beta H),$$
$$\dot{K} = g_H - \exp C, \quad \text{where} \quad g_H = \exp(S_H + \alpha K + (\beta - 1)H).$$

Then $f = (f_K, f_H) = (g_K - \exp C, g_H - \exp C)$, the Jacobian matrix of f is

$$J_f = \begin{pmatrix} (\alpha - 1)g_K & \beta g_K \\ \alpha g_H & (\beta - 1)g_H \end{pmatrix},$$

and the first two assumptions of Olech's theorem hold:

$$\operatorname{tr} J_f = (\alpha - 1)g_K + (\beta - 1)g_H < 0,$$
$$\det J_f = (1 - \alpha - \beta)g_K g_H > 0.$$

The third assumption is also satisfied:

$$\frac{\partial f_K}{\partial K}\frac{\partial f_H}{\partial H} = (\alpha - 1)(\beta - 1)g_K g_H \neq 0, \quad \frac{\partial f_K}{\partial H}\frac{\partial f_H}{\partial K} = \alpha\beta g_K g_H \neq 0.$$

Thus, the unique steady state is globally asymptotically stable!

Global stability in Mankiw-Romer-Weil model III

Going back to the original Mankiw-Romer-Weil model:

$$\dot{k} = s_k k^\alpha h^\beta - c\,k,$$

$$\dot{h} = s_h k^\alpha h^\beta - c\,h.$$

Since the logarithmic coordinates map the positive ortant on IR^2 onto IR^2, the above system has a unique equilibrium in the positive ortant on IR^2, given by

$$k^* = \left(c^{-1} s_k^{(1-\beta)} s_h^\beta\right)^d,$$

$$h^* = \left(c^{-1} s_k^\alpha s_h^{(1-\alpha)}\right)^d,$$

$$d = \frac{1}{1-\alpha-\beta}.$$

This equilibrium is globally asymptotically stable, which makes the economists happy!

PART III

Goodwin model of economic growth

and employment-wage cycles

Goodwin growth model - notation

In 1967 Richard S. Goodwin proposed a model of economic growth showing interaction of income distribution and (un)employment in cycle oscillations. Let

q – aggregate output (production)

k – (homogeneous) capital

ℓ – employment of labor, w – wages,

n – total labor, assumed to grow at rate β,

$n(t) = n(0)\exp(\beta t)$

$e = \ell/n$ – the employment rate

$u := \frac{w\ell}{q}$ – the share of labor in aggregate output q

$a = q/\ell$ – labor productivity,

a is assumed to grow at rate α, $a(t) = a(0)\exp(\alpha t)$

$\sigma = k/q$ – the capital-output ratio, a constant

From the definitions of a and σ

$$q = a\ell = \frac{k}{\sigma}.$$

Goodwin growth model - dynamic equations

Denote

$e = \ell/n$ – the employment rate,

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The postulated dynamic equations of the Goodwin model are

$$\dot{e} = \chi e - \sigma^{-1} ue,$$

$$\dot{u} = -(\gamma + \alpha)u + \rho ue$$

(Lotka-Volterra equations) where γ, ρ *are constants and*

$$\chi = \sigma^{-1} - (\alpha + \beta).$$

Goodwin model - equilibrium and cycles

The equation $\dot{e} = \chi e - \sigma^{-1} u e$ follows from the definitions and assumptions while the second equation $\dot{u} = -(\gamma + \alpha) u + \rho e u$ is a consequence of the Goodwin's postulate (Phillips' curve):

$$\hat{w} = -\gamma + \rho e.$$

and $\hat{u} = \hat{w} - \alpha$.

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$$\hat{w} = -\gamma + \rho e.$$

and $\hat{u} = \hat{w} - \alpha$.
There is a **unique equilibrium point** defined by $\dot{e} = 0 = \dot{u}$:

$$u* = 1 - \sigma(\alpha + \beta),$$

$$e* = \frac{\gamma + \alpha}{\rho}.$$

The equations, as Lotka-Volterra equations, can be integrated and they have a first integral.
The solutions away of the equilibrium **are cycles.**
No asymptotic stability!

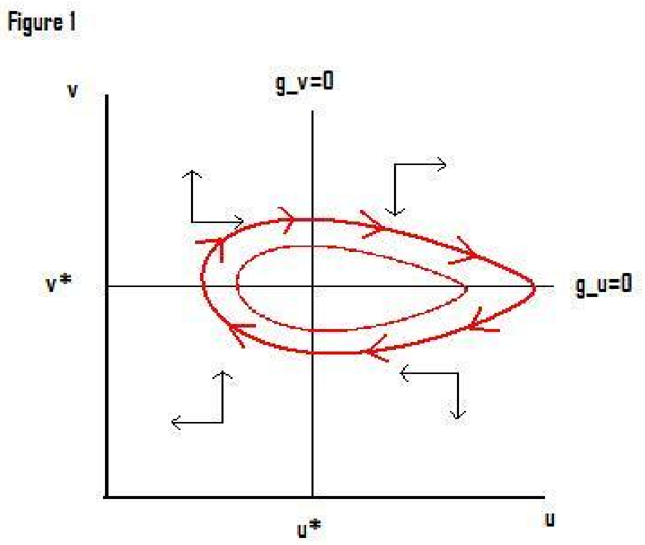
Figure 1
v
g_v=0
v*
g_u=0
u*
u

Modified Goodwin model

The model of Goodwin was modified by Meghnad Desai in 1973 to include the actual inflation and expected inflation. Let

m – money wage

p – price level

Then the real wages w are $w = m/p$ and

$$\dot{w}/w = \hat{w} = \hat{m} - \hat{p}.$$

Then the labor wage share is $u = w/a = m/(pa)$ and

$$\hat{u} = \hat{m} - \hat{p} - \alpha.$$

Desai postulates the equations

$$\hat{m} = -\gamma + \rho e + \eta \hat{p},$$

where $\eta \in [0, 1]$, and the price adjustment equation

$$\hat{p} = \lambda(\log u + \log \pi), \quad \pi > 1.$$

The term $\eta \hat{p}$ represents the expected inflation and η may be called the illusion coefficient.

With the above modifications the first equation of the dynamics does not change

$$\dot{e} = \chi e - \sigma^{-1} ue \qquad (\star)$$

and the second takes the form

$$\dot{u} = -\delta u + \rho ue + \lambda(\eta - 1) u \log u, \qquad (\star\star)$$

where

$$\chi = \sigma^{-1} - (\alpha + \beta),$$

$$\delta = \gamma + \alpha + (1 - \eta)\lambda \log \pi.$$

The new dynamical system $(\star)$, $(\star\star)$ is no more a Lotka-Volterra system. Again, it has a unique equilibrium (e^*, u^*).
The equilibrium is globally asymptotically stable if $\eta < 1$ (inflation illusion). This follows from Olech's Theorem.

In logarithmic coordinates $x = \log e$, $y = \log u$ and $a_0 = \chi$, $a_1 = -\sigma^{-1}$, $b_0 = -\delta$, $b_1 = (\eta - 1)\lambda$, $b_2 = \rho$ equations $(\star)$, $(\star\star)$ become

$$\dot{x} = a_0 + a_1 \exp y$$

$$\dot{y} = b_0 + b_1 y + b_2 \exp x.$$

Jacobian matrix

$$\begin{pmatrix} 0 & a_1 \exp y \\ b_2 \exp x & b_1 \end{pmatrix}.$$

If $\eta < 1$ then all three conditions of Olech's Theorem hold:

$$\mathrm{tr}\, J = b_1 = (\eta - 1)\lambda < 1$$

$$\det J = -a_1 b_2 \exp x \exp y = \sigma^{-1} \rho \exp x \exp y \;>\; 0$$

$$\frac{\partial f_1}{\partial y} \frac{\partial f_2}{\partial x} = a_1 b_2 \exp y \exp x \neq 0.$$

PART IV
Common work with Philip Hartman

During the same stay in RIAS in the academic year 1960-1961 Olech wrote a common paper with Philip Hartman. Two simplest results in that paper concern a system

$$\Sigma: \quad \dot{x} = f(x), \quad x \in IR^n,$$

with f of class C^1.

Theorem

Denote $J(x) = \partial f / \partial x(x)$ and assume:
(i) $x_0 = 0$ is a locally asymptotically stable equilibrium of Σ,

$$(ii) \quad \text{for all } \; x, y \in IR^n: \quad \langle J(x)y, y\rangle \leq 0, \quad \text{if} \quad \langle f(x), y\rangle = 0.$$

Then $x_0 = 0$ is globally asymptotically stable.

Theorem

Denote $J_S = (J + J^T)/2$ - the symmetrized Jacobian. Assume:
(i) $x_0 = 0$ is a locally asymptotically stable equilibrium of Σ,

$$(ii) \qquad \int_0^\infty \min_{|x|=s} |f(x)| ds = \infty,$$

$$(iii) \qquad \lambda_1(x) + \lambda_2(x) \leq 0, \quad x \in IR^n,$$

for the eigenvalues $\lambda_1(x) \geq \cdots \geq \lambda_n(x)$ of $J_S(x)$.
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for the eigenvalues $\lambda_1(x) \geq \cdots \geq \lambda_n(x)$ *of* $J_S(x)$*.*
Then $x_0 = 0$ *is globally asymptotically stable.*

Note that if $n = 2$ then (iii) is equivalent to

$$\mathrm{tr}\, J(y) \leq 0, \quad y \in IR^n.$$

Symposium on
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* To be confirmed

Preliminary programme:

Zvi Artstein (Weizmann Institute)
Olech's extremals in the singular perturbations limit

Alberto Bressan (Penn State University)
Recent trends in differential inclusions

Pavol Brunovsky (Comenius University)
Infinite horizon optimal control, dichotomy and closed range

Arrigo Cellina (University of Milano-Bicocca)
On the regularity of solutions to variational problems

Asen Dontchev (Math. Reviews and University of Michigan)
The four theorems of Laurence M. Graves (to be confirmed)

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