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On the numerical solution of an ODE eigenproblem using an orthogonal expansion

We treat here a differential equation of an arbitrary fair order m

$$\sum_{r=0}^{m} a_r(x)y^{(r)}(x) = \lambda \sum_{r=0}^{p} b_r(x)y^{(r)}(x) \quad p \leqslant m \leqslant n \tag{1}$$

with the boundary conditions:

$$y^{(r)}(a) = 0 \quad \underset{0 \leqslant r \leqslant 0.5m-1}{\forall} \tag{2}$$

$$y^{(r)}(b) = 0 \quad \underset{0 \leqslant r \leqslant 0.5m-1}{\forall} \tag{3}$$

where we expand the unknown function $y(x)$ by orthogonal polynomials

$$y(x) = \sum_{j=0}^{n} y_j \phi_j(x) \tag{4}$$

$$y_j = \frac{(y, \phi_j)_{L^2[a,b]}}{||\phi_j||^2_{L^2[a,b]}} \quad \underset{0 \leqslant j \leqslant n}{\forall} \tag{5}$$

After setting these expansions to the differential equation (1) and using the Least Squares Method we reduce the problem to a linear algebraic problem easy to treat numerically. First we obtain by interpolation for given number values of λ the secular polynomial whose roots are the approximations of the searched eigenvalues, and after, for each eigenvalue a homogeneous equations system of order at least $m + 1$, which has the nonzero solution being the part of orthogonal expansion coefficients and the remaining ones are obtained from the given boundary conditions. The obtained orthogonal expansions are the approximations of the eigenfunctions for determined eigenvalues.

Remark: The LSM was shown in [1, 2, 3], but the approach was different of that presented above.

References

[1] L. Collatz. *Eigenwertaufgaben mit technischen Anwendungen.* Geest und Portig, Leipzig, 1949.
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[3] M. Nasta. Atti d. R. Accademia Nazionale dei Lincei (6) 12 (1930), 212.