

Moshe Goldberg

Department of Mathematics, Technion – Institute of Technology, Haifa, Israel

Stability of the θ -Method for Parabolic Initial-Value Problems

The purpose of this talk is to discuss stability criteria for the θ -Method for parabolic initial-value problems of the form

$$\frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial t} = \sum_{1 \leq p \leq q \leq s} A_{pq} \frac{\partial^2 \mathbf{u}(\mathbf{x}, t)}{\partial x_p \partial x_q} + \sum_{1 \leq p \leq s} B_p \frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial x_p} + C \mathbf{u}(\mathbf{x}, t),$$

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{f}(\mathbf{x}), \quad \mathbf{x} = (x_1, \dots, x_s) \in \mathbb{R}^s, \quad 0 \leq t \leq T,$$

where A_{pq} , B_p , and C are constant matrices. This method is a well-known family of finite-difference approximations depending on a parameter θ , $0 \leq \theta \leq 1$, which includes the Euler Scheme ($\theta = 0$), the Crank–Nicholson Scheme ($\theta = \frac{1}{2}$), and the Backward Euler Scheme ($\theta = 1$). We shall deal with two cases, the classical case where the leading matrix coefficients A_{pq} are Hermitian, and the less conventional case where the A_{pq} are triangular. In the first case, our initial-value problem provides, for example, a model for diffusion of several species where the first- and zero-order terms represent transport and chemical reactions. The second case arises, for instance, in connection with heat and mass transfer with Soret and Dufour cross-effects.