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Asymptotically periodic solutions to difference equations of Sturm–Liouville type

In this paper we consider the second order difference equation of the form

$$\Delta(r_n\Delta x_n) = a_n f(x_{n-\tau}) + b_n \tag{E}$$

where $r_n, a_n, b_n \in \mathbb{R}$, $r_n \neq 0$, $f : \mathbb{R} \to \mathbb{R}$, $\tau \in \mathbb{Z}$.

Note that equation (E) generalizes the well known Sturm–Liouville difference equation

$$\Delta(r_n\Delta x_n) = a_n x_{n+1}, \tag{1}$$

which has arisen in the study of various fields, such as electrical circuit analysis, matrix theory, control theory and discrete variational theory. Equation (1) has been extensively investigated by many authors. Schmeidel and Zbąszyniak (2012) studied the following special case of (E)

$$\Delta(r_n\Delta x_n) = a_n f(x_{n+1})$$

by using Darbo's fixed point theorem. They obtained sufficient conditions for the existence of an asymptotically periodic solution. In this paper, we improve and generalize this result. We present sufficient conditions under which equation (E) possesses uncountably many asymptotically periodic solutions. Moreover, we also give sufficient conditions under which every bounded solution of (E) is asymptotically periodic.